Optically Induced Coherence Effects in Quantum Metamaterials
Self-induced Transparency and Dicke Superradiance

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Outline

Abstract

Introduction

SIT in QMM

Conclusion

Optically Induced Coherence Effects in Quantum Metamaterials
Overview of research

- Propagation of the Electromagnetic radiation in quantum metamaterials (QMM)
  - QMM ⇔ one-dimensional chain of the large number of aligned identical superconducting charge qubit (SCCQB)
    - Rakhmanov, Zagoskin, Savel’ev, Nori, PRB 77 144507 (2008)

- Theoretical prediction of two remarkable coherent optical phenomena:
  - Self-induced transparency (SIT)
    - Two–photon self–induced transparency (TPSIT)
  - Dicke superradiance
Remainder: SIT and Superradiance

- Group of N (N \gg 1) Absorbers or Emitters—“atoms” in ground or excited state—interact with a common light field whose wavelength is much greater than the inter–atomic separation.
- SIT—Material which normally absorbs light becomes completely transparent to a short-duration light pulse whose intensity exceeds some critical value.
- Collective emission of the high intensity coherent pulse \( I \sim N^2 \)
- Expected behaviour \( I \sim N \)

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Remainder: SIT

- It is the consequence of the reemission of absorbed radiation in phase with the driving optical field.
- large pulse
- short duration $\tau_p \gg \tau_S$
- single pulse may excite atom and produce stimulated de-excitation

Area theorem: SIT may occur only if pulse "area" satisfy

$$\int dtE^s(t) = 2\pi N, \; N = 1, 2, 3... \; s = 1, 2...$$
SIT–Features

- Lossless propagation: input energy is equal to the energy of output pulse
  - there is no spontaneous emission: pulse duration is much shorter than the life–time of atom in the excited state
- Reshaping of the pulse: the output pulse may be reshaped by the medium if the input pulse is not SYMMETRIC HYPERBOLIC SECANT
- Pulse is slowed down
  - As light pulse passed through the absorber, it is delayed.
  - for the absorber of length \( l \) delay time is: \( \tau_D = \frac{l}{v} - \frac{l}{c} \)
Manipulation of light (Yet another?)

- Slowing down of light.
- QMMs offer controllable, enhanced slowing down of light

The experimental confirmation of these effects in QMM may open a new pathway to potentially powerful quantum computing.
Abstract

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The Model: Experimental setup

- QMM: Large number of identical SCCQB embedded inside the massive superconductor
### Theoretical model

#### Variables

- \( a_n = 2\pi DA_{x,n}/\Phi_0 \) - dimensionless EM potential.
- \( \varphi_n \) the superconducting phase on \( n \)th island.

\[
H = \sum_n \left[ \frac{\varphi_n^2}{2} - 2 \cos \varphi_n \right] + \sum_n \left[ \frac{\dot{\varphi}_n^2}{2} + \beta^2 (\alpha_{n+1} - \alpha_n)^2 \right] + \sum_n [2 \cos \varphi_n (1 - \cos \alpha_n)],
\]

#### Energy parameters

- Energy unit \( E_J = \Phi_0 I_c / (2\pi c) \)–Josephson energy,
- \( \omega_J^2 = 2eI_c / \hbar C \) Josephson frequency
- \( \beta^2 = (8\pi DE_J)^{-1} (\Phi_0 / 2\pi)^2 \) -dimensionless speed of light,
- \( \Phi_0 = \hbar / (2e) \) is the magnetic flux quantum,
- \( I_c \) and \( C \) is the critical current and capacitance, respectively, of the JJs,
- \( D \) is the separation of the superconducting electrodes of the waveguide,
- dimensionless time \( \tau = \omega_J t \), with \( \omega_J = eI_c / (\hbar C) \) and \( t \) being the temporal variable in natural units.

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Two-level truncation

- SCCQB Eigenvalues \((E_p, n)\) and eigenfunctions \(\gamma_p(\varphi_n)\) are given as solutions of the Mathieu equation \((-\partial^2/\partial \varphi_n^2 + E_{p,n} - 2 \cos \varphi)\gamma_p(\varphi_n) = 0\)

- In the most cases of the interest only two lowest levels are important \(E_p\) \((p = 0, 1)\)

- Effective two level model

\[
H = \sum_{n,p} E_p(n) a_{n,p}^\dagger a_{n,p} + \sum_n \left[ \dot{\alpha}_n^2 + \beta^2(\alpha_{n+1} - \alpha_n)^2 \right] + 4 \sum_{n,p,p'} V_{p,p'}(n) a_{n,p}^\dagger a_{n,p'} \sin^2 \frac{\alpha_n}{2}
\]

\[
V_{p,p'}(n) = \int d\varphi_n \gamma_p^*(\varphi_n) \cos \varphi_n \gamma_p(\varphi_n)
\]

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EM radiation in QMM: Evolution equations

- Schrödinger equation (SE) for qubit: We took that qubit is in the superposition state

\[ |\Psi_n\rangle = \sum_{n,p=0,1} \Psi_{n,p}a_p^+|0\rangle \]

\[ i\dot{\Psi}_{n,p} = \epsilon_p \Psi_{n,p} + \frac{4E_J}{\hbar \omega_J} \sum_{p'} V_{p,p'}(n)\Psi_{n,p'} \sin^2 \frac{\alpha_n}{2} \]

- Maxwell equation for EM wave.

\[ \ddot{\alpha}_n = \beta^2 (\alpha_{n+1} + \alpha_{n-1} - 2\alpha_n) - 2 \frac{E_J}{\hbar \omega_J} \sum_{p,p'} V_{p,p'} \psi_{n,p}^* \psi_{n,p'} \sin \alpha_n, \]
Approximations and Analytic solutions

- Linearized model: small intensity of EM radiation $\alpha_n \ll 1$
- Slowly varying envelope approximation
- Exact resonance of EM radiation and qubit
- Continuum approximation
Bloch variables

- Maxwell–Bloch equations for amplitude of EM field \( \alpha_n(t) \)
  and Components of the Bloch vector

\[
\ddot{\alpha}_n(t) - \beta^2_0(\alpha_{n+1} + \alpha_{n-1} - 2\alpha_n) - \left[ \frac{\Omega^2}{2} + \mu R_3 + DR_3 \right] \alpha_n = 0.
\]

\[
\begin{align*}
R_1 &= |\psi_1|^2 - |\psi_0|^2, & \dot{R}_1 &= -\mu \alpha_n^2 R_2, \\
R_2 &= i(\psi_1^*\psi_0 - \psi_0^*\psi_1), & \dot{R}_2 &= -[\Delta + \Omega \alpha_n^2] R_3 + \mu \alpha_n^2 R_1 \\
R_3 &= \psi_1^*\psi_0 + \psi_0^*\psi_1, & \dot{R}_3 &= [\Delta + \Omega \alpha_n^2] R_2.
\end{align*}
\]

where \( V_{01} = V_{10}, D = (V_{11} - V_{00})E_J/(2\hbar\omega_J), \Omega^2 = (V_{11} + V_{00})E_J/(2\hbar\omega_J), \)

\( \mu = V_{10}E_J/(\hbar\omega_J), \) and \( \Delta = \epsilon_1 - \epsilon_0 \equiv (E_1 - E_0)E_J/(\hbar\omega_J). \)
Slowly varying envelope approximation

- Introduction of slow phase and envelope

\[ \alpha(x, t) = \varepsilon(x, t) \cos \Psi(x, t) \]
\[ \Psi(x, t) = kx - \omega t + \varphi(x, t) \]

- Rotation in abstract space, around \( R_y \) axis.

\begin{align*}
R_x &= r_x \cos 2\Psi + r_y \sin 2\Psi \\
R_y &= r_y \cos 2\Psi - r_x \sin 2\Psi
\end{align*}
Truncated system

\begin{align*}
\dot{\epsilon} + \frac{\beta^2 k}{\omega} \epsilon_x &= - \frac{\hbar \omega_J}{E_J} \frac{\mu \varepsilon}{2 \omega} r_y, \\
\dot{\phi} + \frac{\beta^2 k}{\omega} \phi_x &= - \frac{\hbar \omega_J}{E_J} \frac{D}{\omega} R_z,
\end{align*}
\begin{align*}
\dot{r}_x &= -(\Delta - 2 \omega + 2 \dot{\phi} + D \varepsilon^2) r_y, \\
\dot{r}_y &= (\Delta - 2 \omega + 2 \dot{\phi} + D \varepsilon^2) r_x - \frac{\mu \varepsilon^2}{2} R_z, \\
\dot{R}_z &= \frac{\mu \varepsilon^2}{2} r_y.
\end{align*}
Resonant propagation

- truncated system posses soliton like solutions under the resonance and transparency conditions

\[ \Delta = 2\omega, \ k = \pm \frac{\sqrt{\omega^2 - \Omega^2}}{\beta} \]

- lorentzian pulse

\[ \varepsilon = \frac{\varepsilon_0}{\sqrt{1 + \frac{\tau^2}{\tau_p^2}}} \]

\[ \varepsilon_0^2 = -\frac{4R_0\hbar\omega_J v}{E_J(1 + \gamma^2)\omega(c - v)}, \]

\[ \tau_p = -\frac{E_J\omega(c - v)}{R_0\mu\hbar\omega_J v} \sqrt{1 + \gamma^2}, \ \gamma = \frac{4D}{\mu} \]
Conclusions

\[ v = \beta \sqrt{1 - \frac{\hbar \omega_J}{E_J} \left( \frac{\Omega}{\omega} \right)^2 \left[ 1 \pm \frac{4\sigma^2 \hbar \omega_J}{c_0^2 \omega E_J} \right]^{-1}}, \]

\[ v = \beta \sqrt{1 - \frac{\hbar \omega_J}{E_J} \left( \frac{\Omega}{\omega} \right)^2 \left[ 1 \pm \frac{\tau \mu \sigma \hbar \omega_J}{\omega E_J} \right]^{-1}}, \] \hspace{1cm} (3.1)

For absorbing QMMs we deduce from Eq. (3.1) that the more intense pulse propagate faster and that there exist an upper bound on the pulse velocity

\[ v_0 = \beta \sqrt{1 - 2 \frac{V_{11} + V_{00}}{(E_1 - E_0)^2} \left( \frac{\hbar \omega_J}{E_J} \right)^2}, \] \hspace{1cm} (3.2)
results

Figure: Pulse profile (green) and population inversion ($R_z$) versus dimensionless time for a few values of $\gamma$: blue–population inversion for absorbing medium ($r_z(\infty) = -1$), red–population inversion for radiating medium ($r_z(\infty) = 1$).
results

Figure: The velocity of the SIT pulse in units of $\beta$ as a function of the pulse amplitude $\epsilon_0$, for absorbing (solid curves) and amplifying (dashed curves) quantum metamaterials from Nikos Lazarides, George Tsironis, and Zoran Ivić.
Influence of the initial conditions

Figure: Illustration of the influence of the initial conditions on the SIT in QMM

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Figure: Illustration of the influence of the initial conditions on the SIT in QMM.

\[ r_z(-\infty) = \pm \frac{\gamma}{1 + \gamma^2}^{1/2} \equiv \pm 0.76822 \]

\[ r_z(-\infty) = \pm 1 \]

\[ \frac{\epsilon^2}{\epsilon'}^2 \]

\[ \frac{\tau}{\tau'} \]

Optically Induced Coherence Effects in Quantum Metamaterials
Numerical validation

almost coherent pulse propagation for substantial time intervals
Optically Induced Coherence Effects in Quantum Metamaterials
Conclusions

- SIT and superradiance may, in principle, appear in QMM,
- We believe that the predicted effects may appear not only in charge qubits based QMM,
- The benefit–manipulation of light by means of:
  - Tuning of the parameters of the QMM,
  - Choice of the initial conditions
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