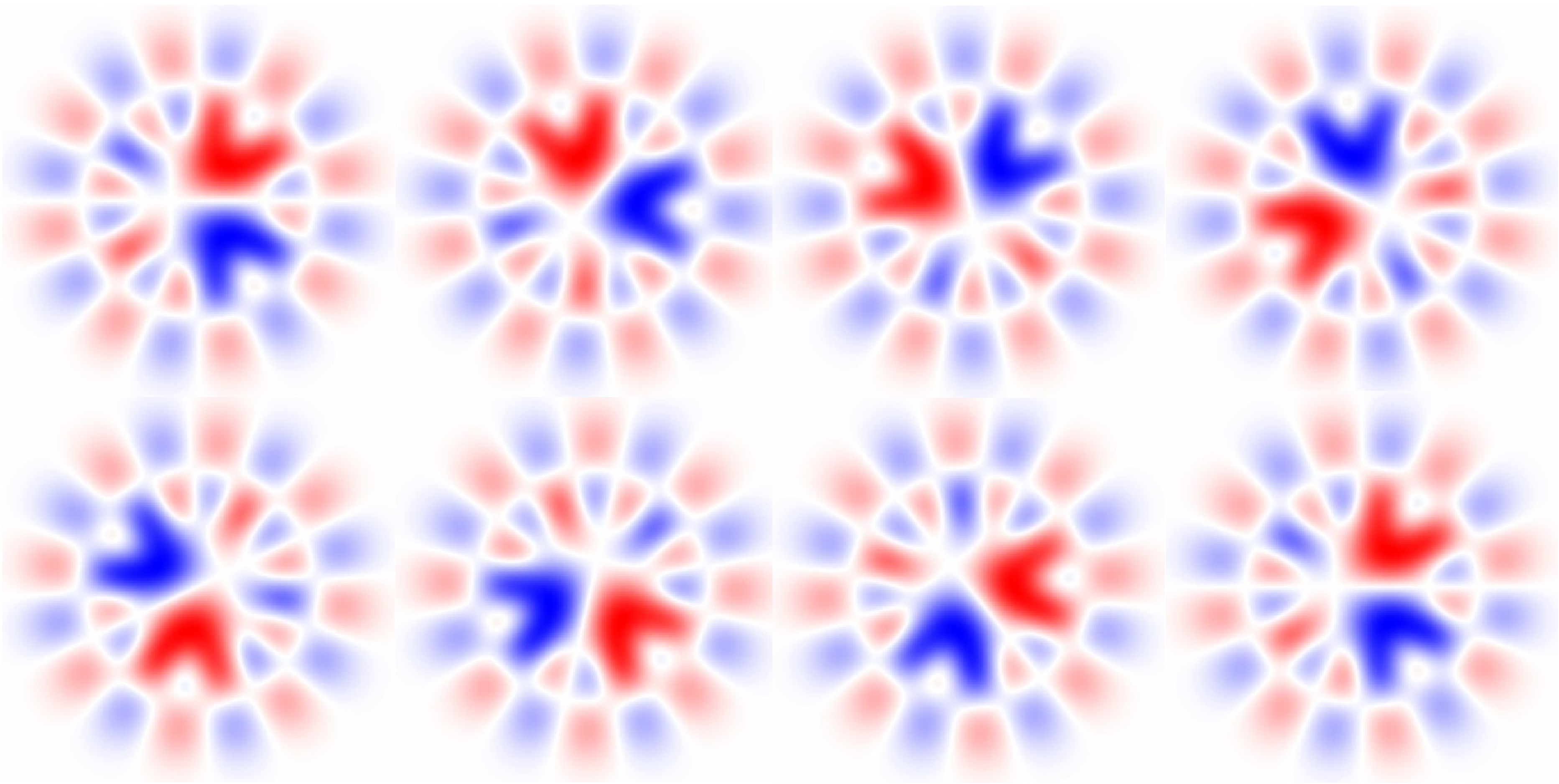
Universiteit Leiden
Leiden Observatory

Galaxy Image Modeling Using Shapelets and Sparse Techniques



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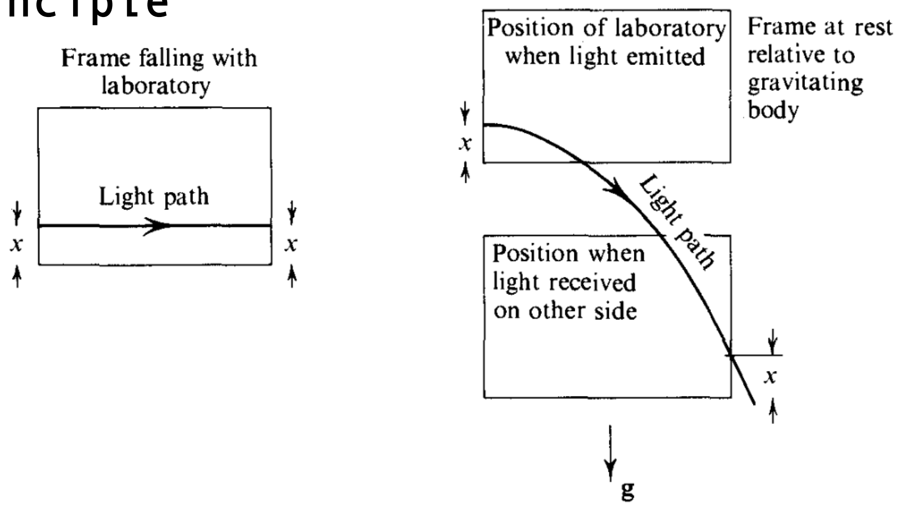
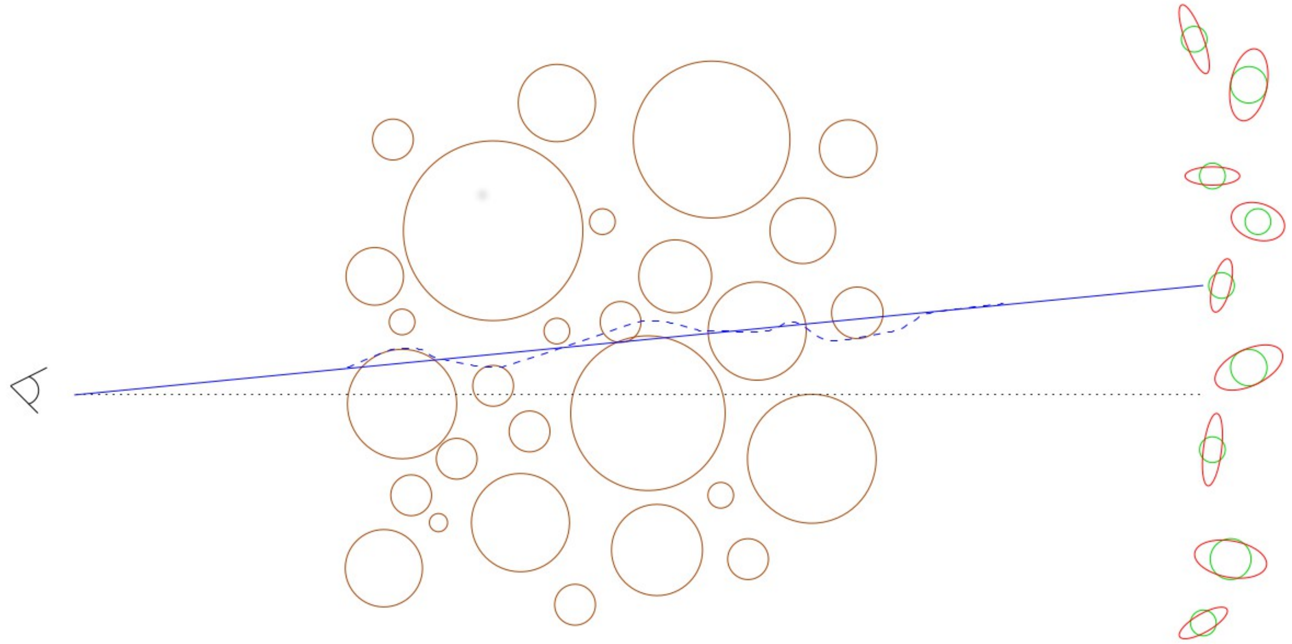
Supervisor:
Arun Kannawadi

Motivation

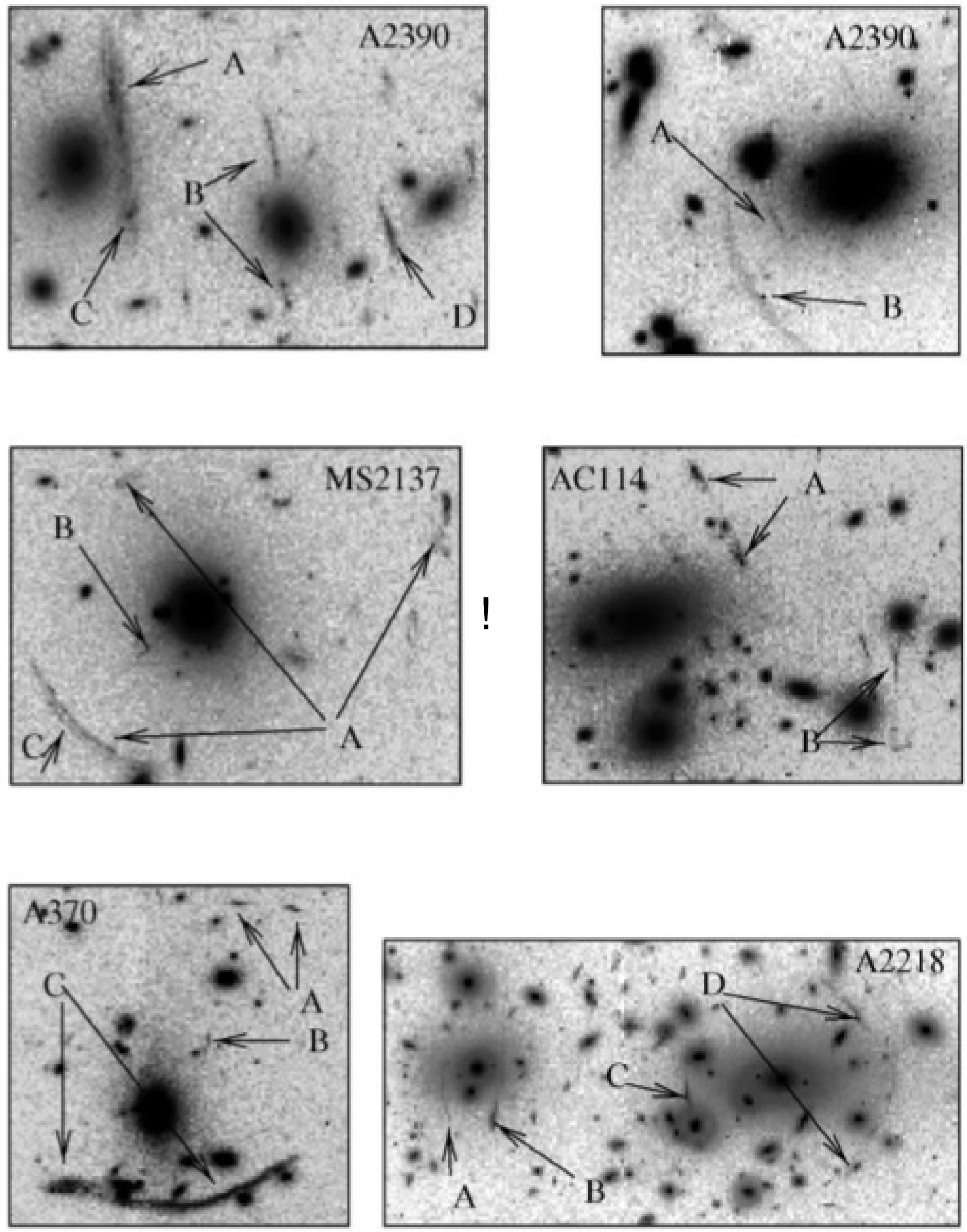
Equivalence principle



Applied to cosmological models

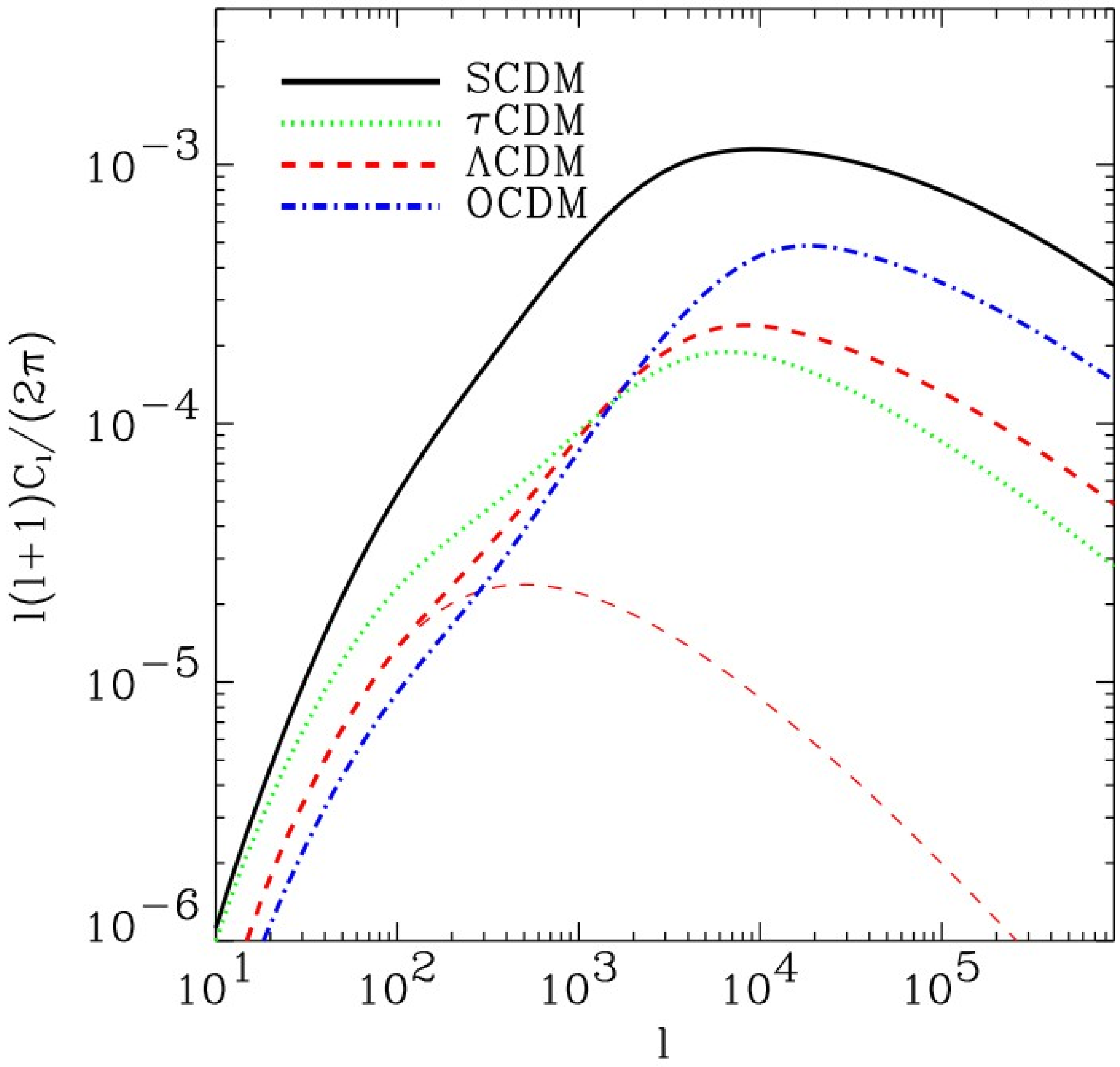


Motivation



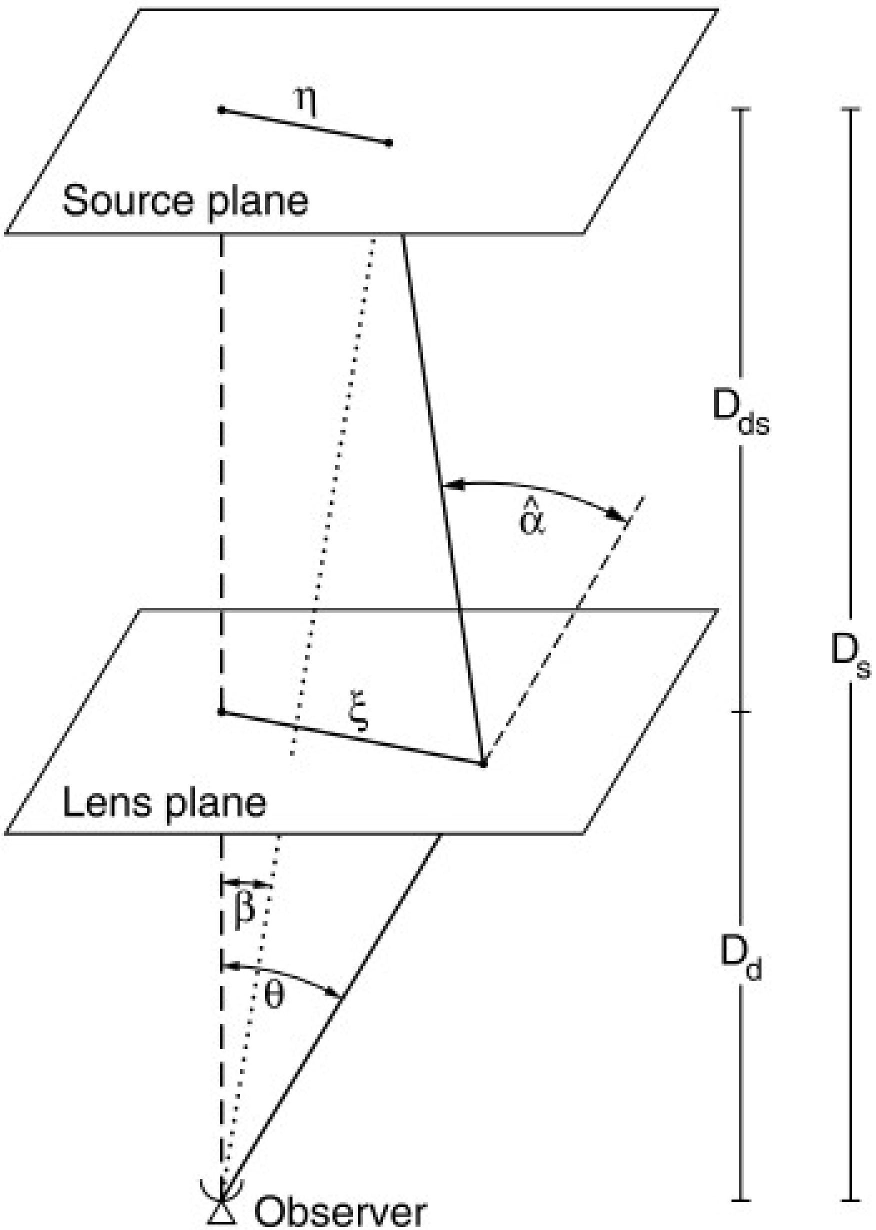
Mellier Y.. ARAA. 1999

Motivation



Refregier A., ARAA, 2003

More precisely

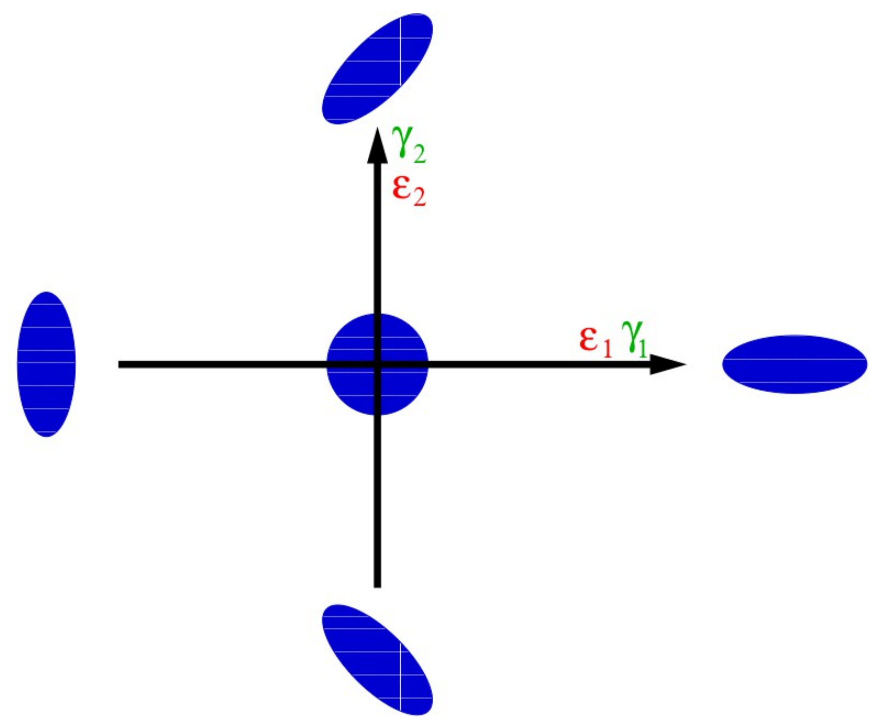


Surface brightness

$$I(\boldsymbol{\theta}) = I^{(\mathrm{s})}[\boldsymbol{\beta}(\boldsymbol{\theta})]$$

Jacobian of transformation

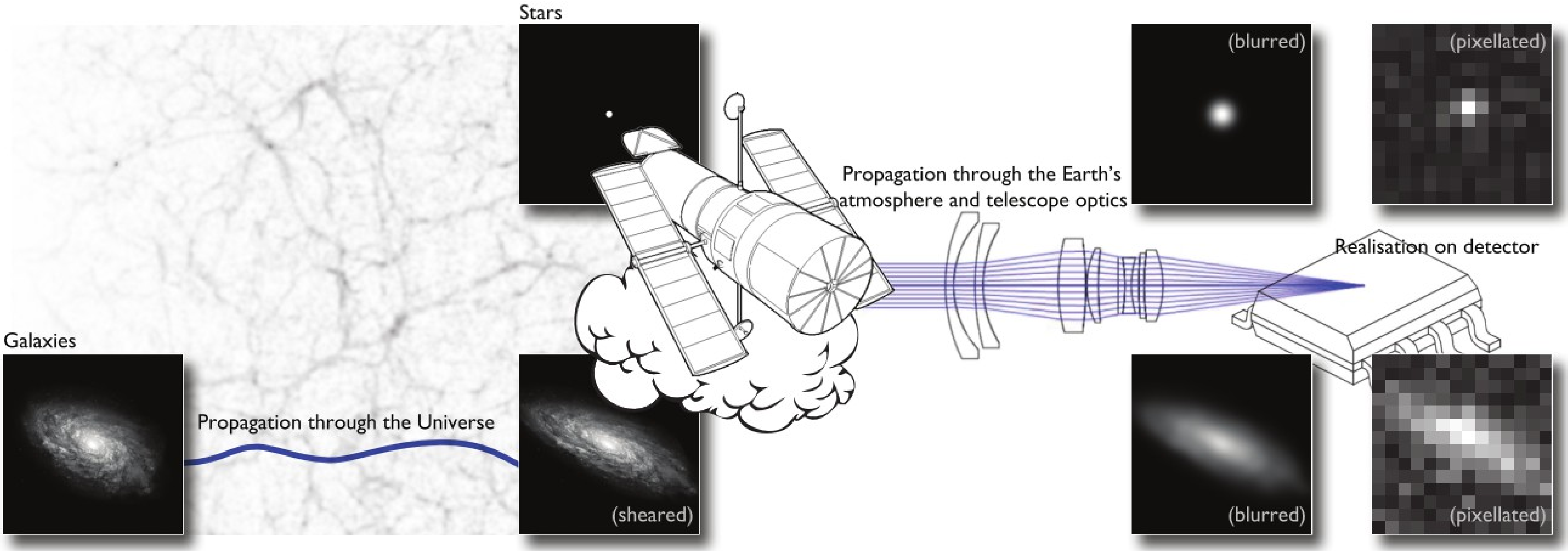
$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \left(\delta_{ij} - \frac{\partial^2 \psi(\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j}\right) = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$



A2218 cluster at z = 0.175 - HST

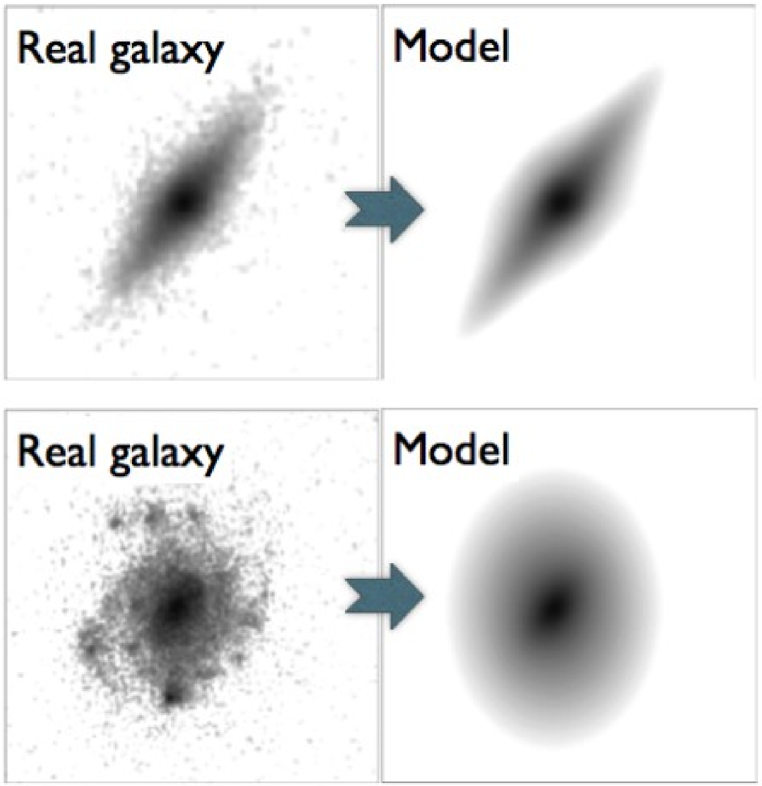
It's the ellipticity averaged over the ensemble of galaxies that is
Important for the tidal gravitational field

Weak lensing - difficulties

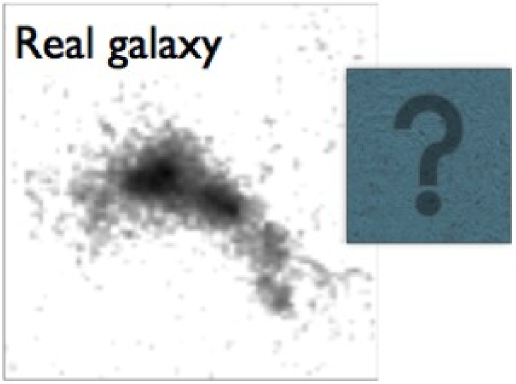


- Signal is very weak, hard to distinguish from the observational distortions
- Ellipticities are nonlinear functions of the measured brightness, and hence biased

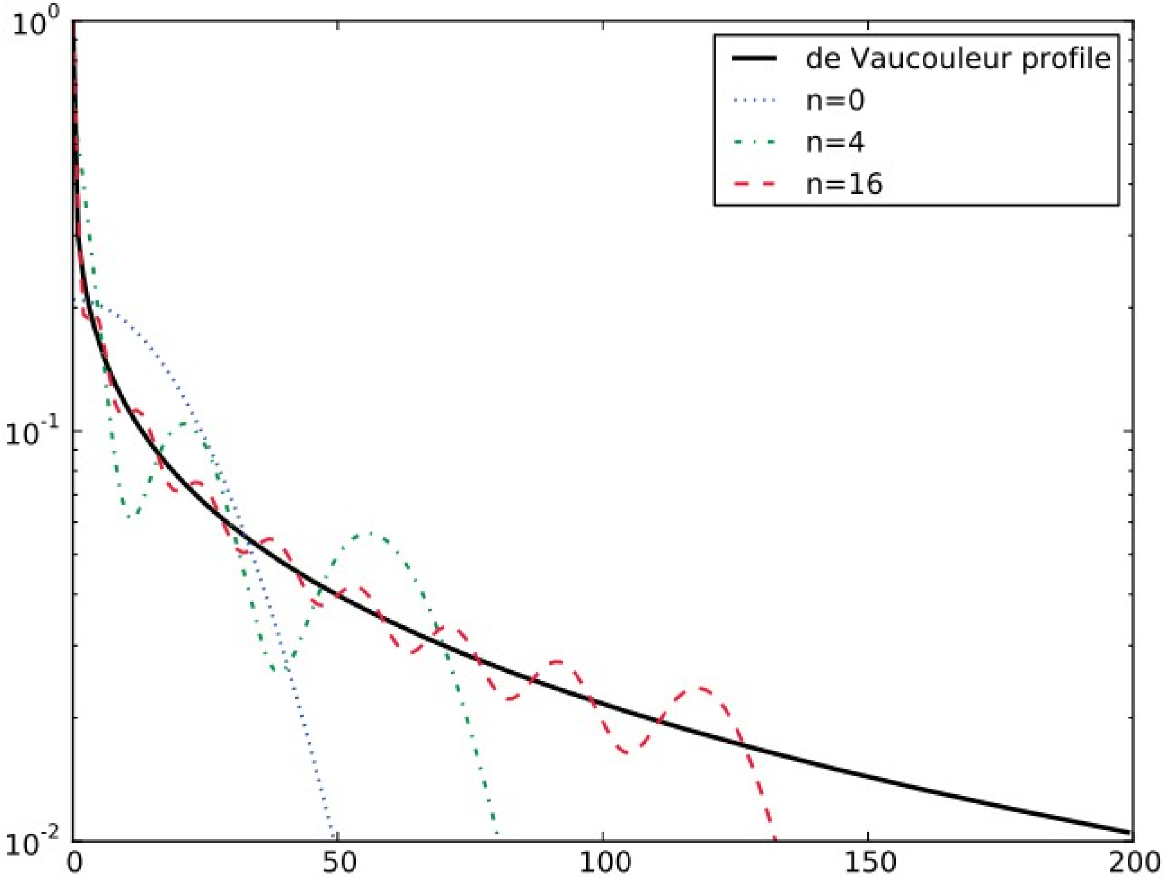
Solution is doing simulations!



Shapelets are very good at capturing irregularly shaped galaxies

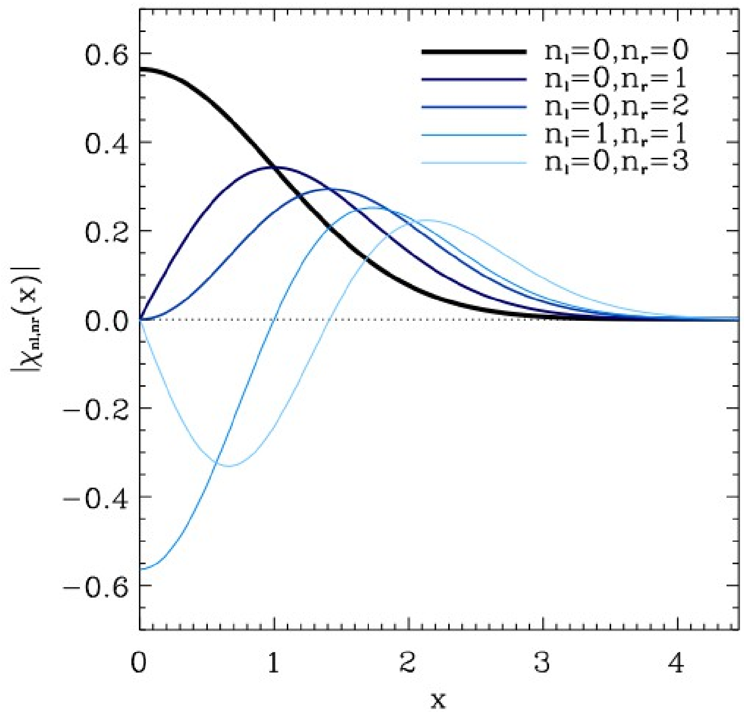


Usually **Sersic** profiles are used for modeling

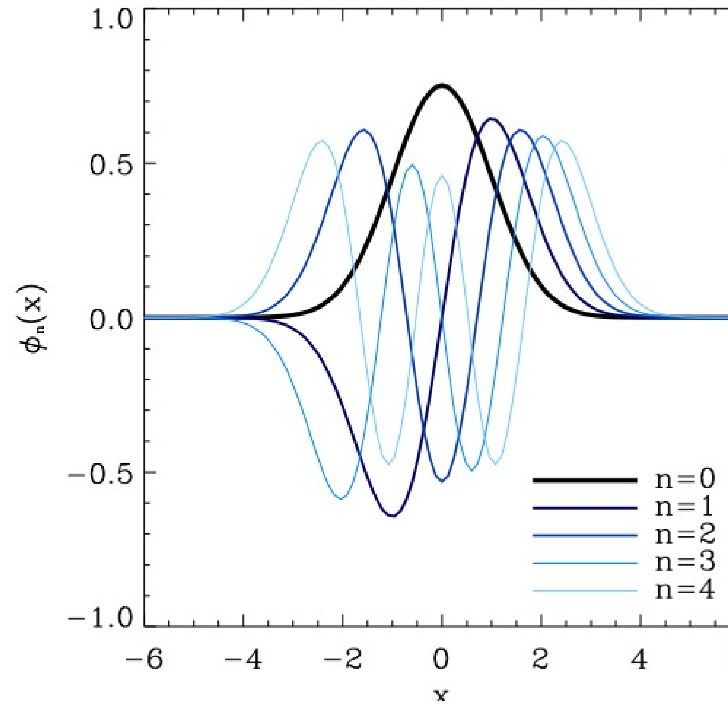


Shapelets - theory

Polar



Cartesian



$$\chi_{n_l,n_r}(x,\varphi) = [\pi n_l! n_r!]^{-\frac{1}{2}} H_{n_l,n_r}(x) e^{-x^2/2} e^{i(n_r - n_l)\varphi}$$

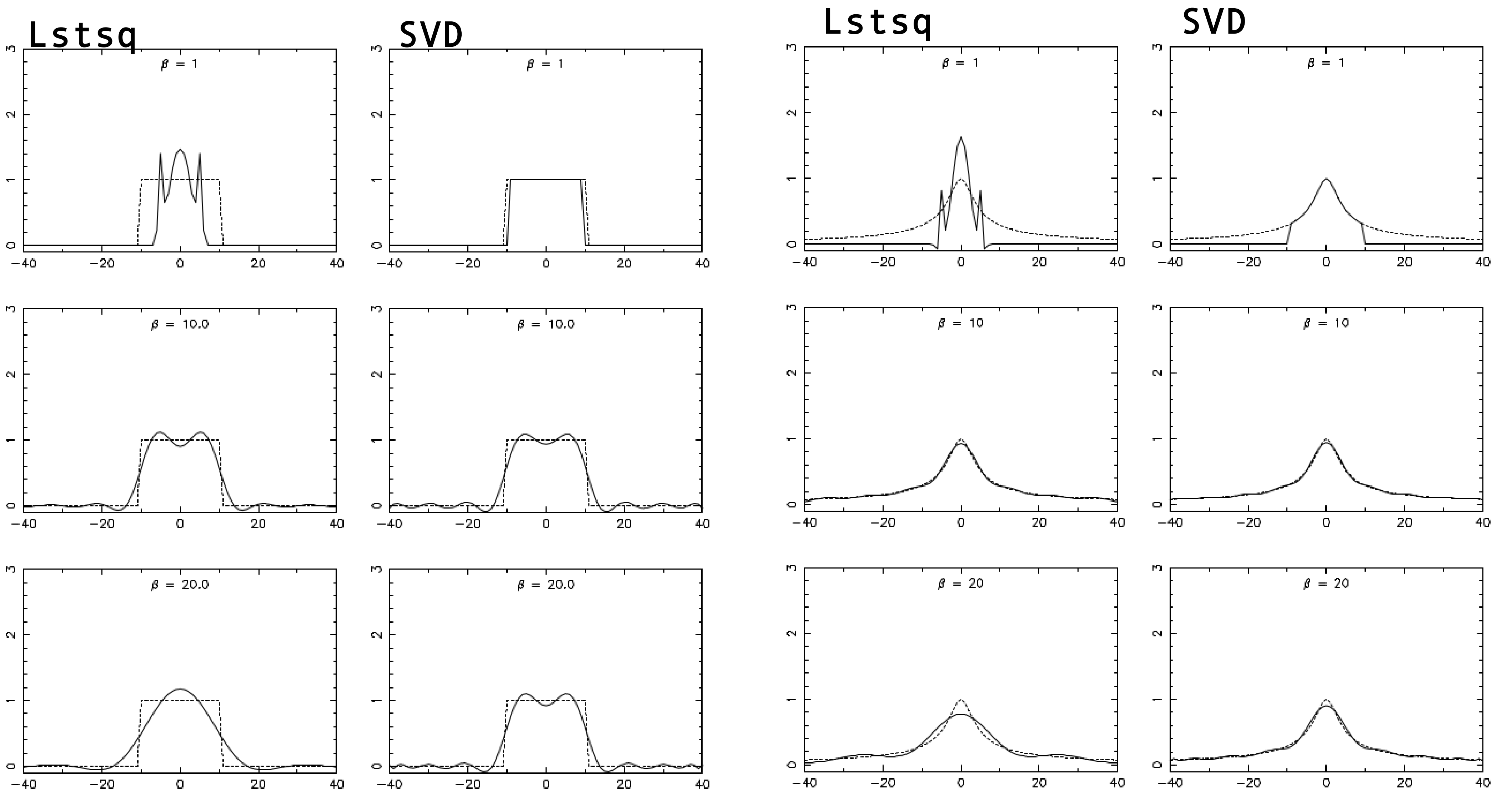
$$\phi_n(x) \equiv \left[2^n \pi^{\frac{1}{2}} n!\right]^{-\frac{1}{2}} H_n(x) e^{-\frac{x^2}{2}}$$

$$A_{n_l,n_r}(x,\varphi;\beta) = \beta^{-1} \chi_{n_l,n_r}(\beta^{-1}x,\varphi)$$

$$B_n(x;\beta) \equiv \beta^{-\frac{1}{2}} \phi_n(\beta^{-1}x)$$

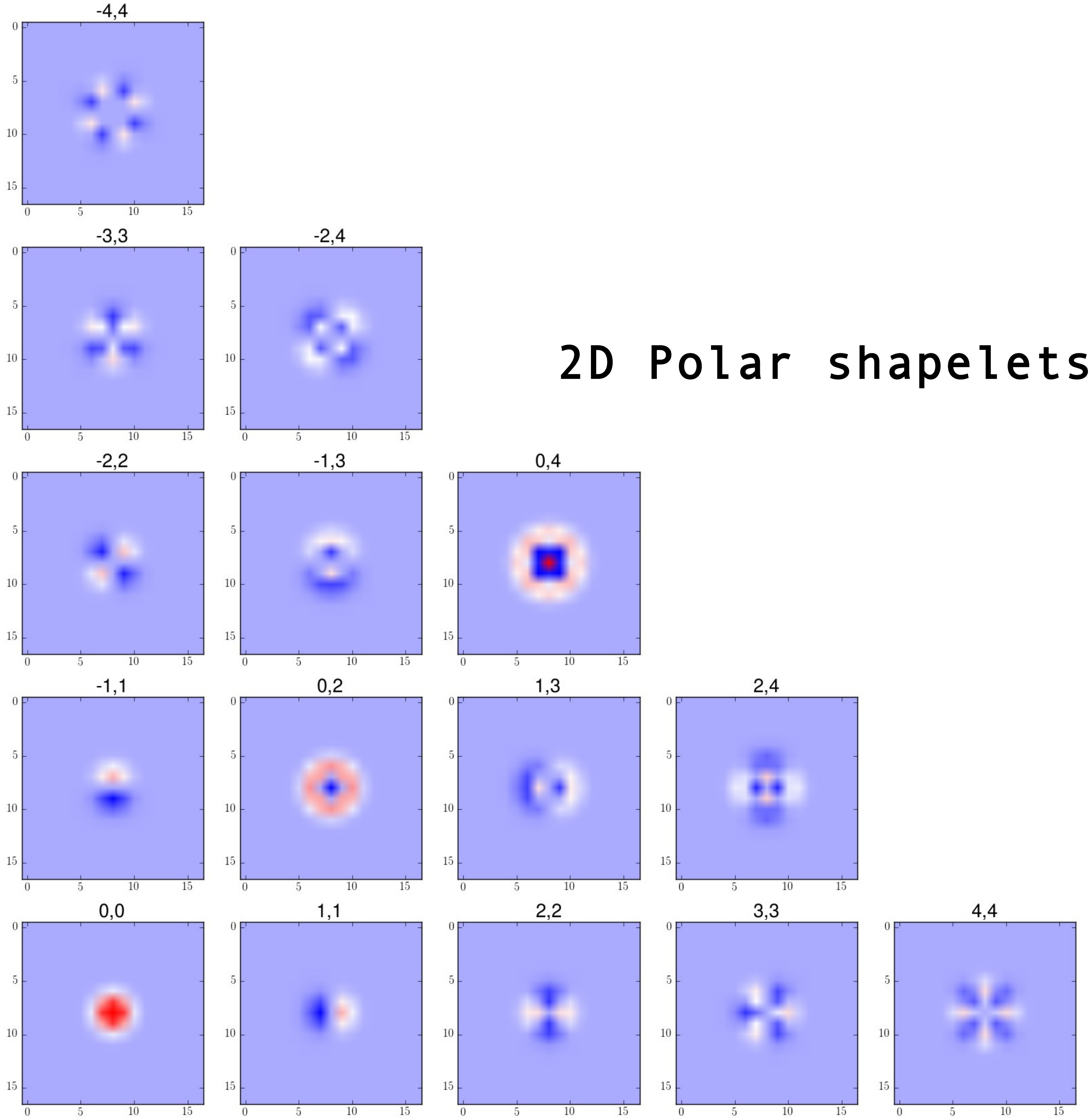
Refregier A., MNRAS, 2001

Example of 1D shapelets decomposition

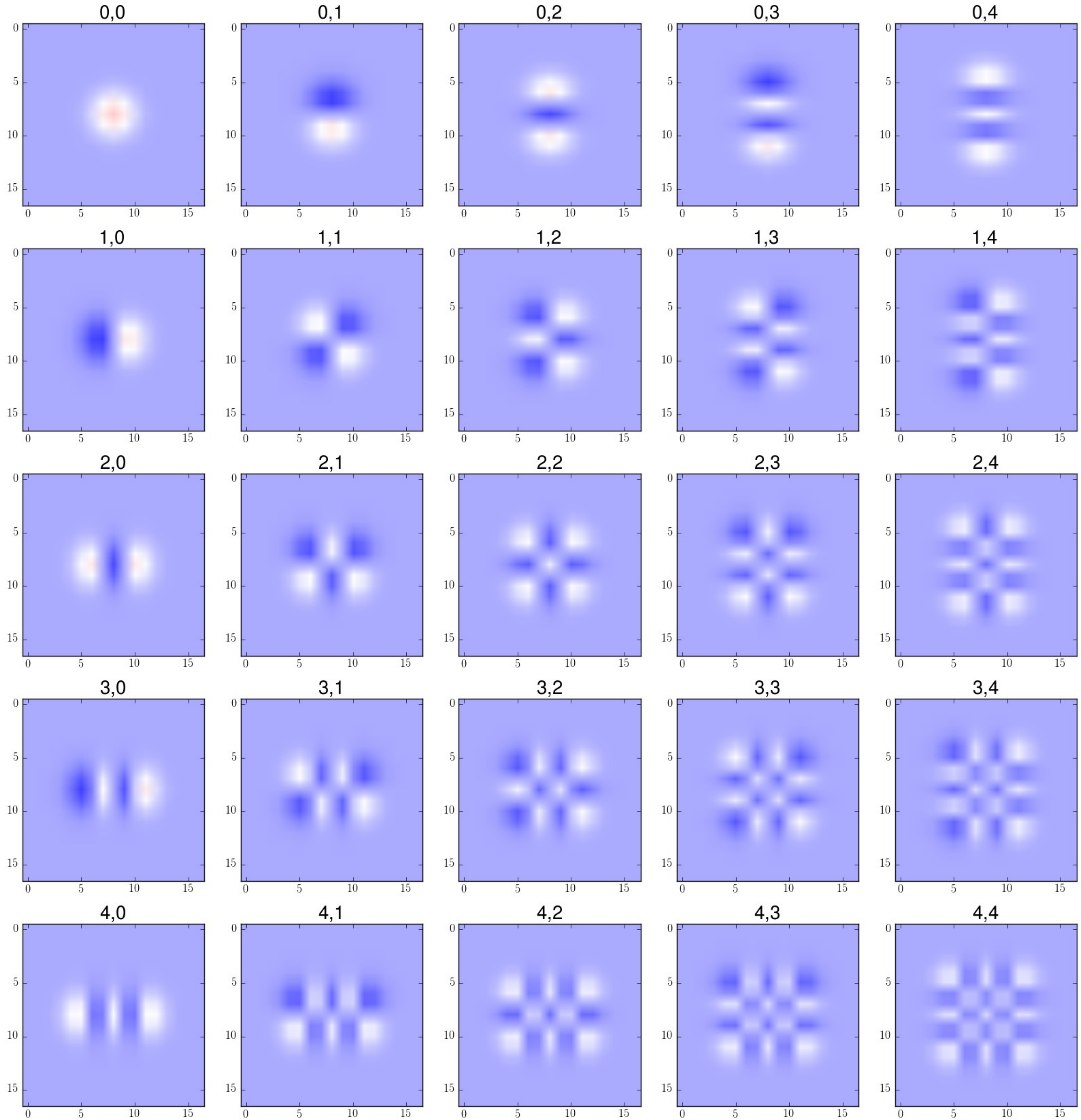


Berry R H et al MNRAS 2004

2D Polar shapelets

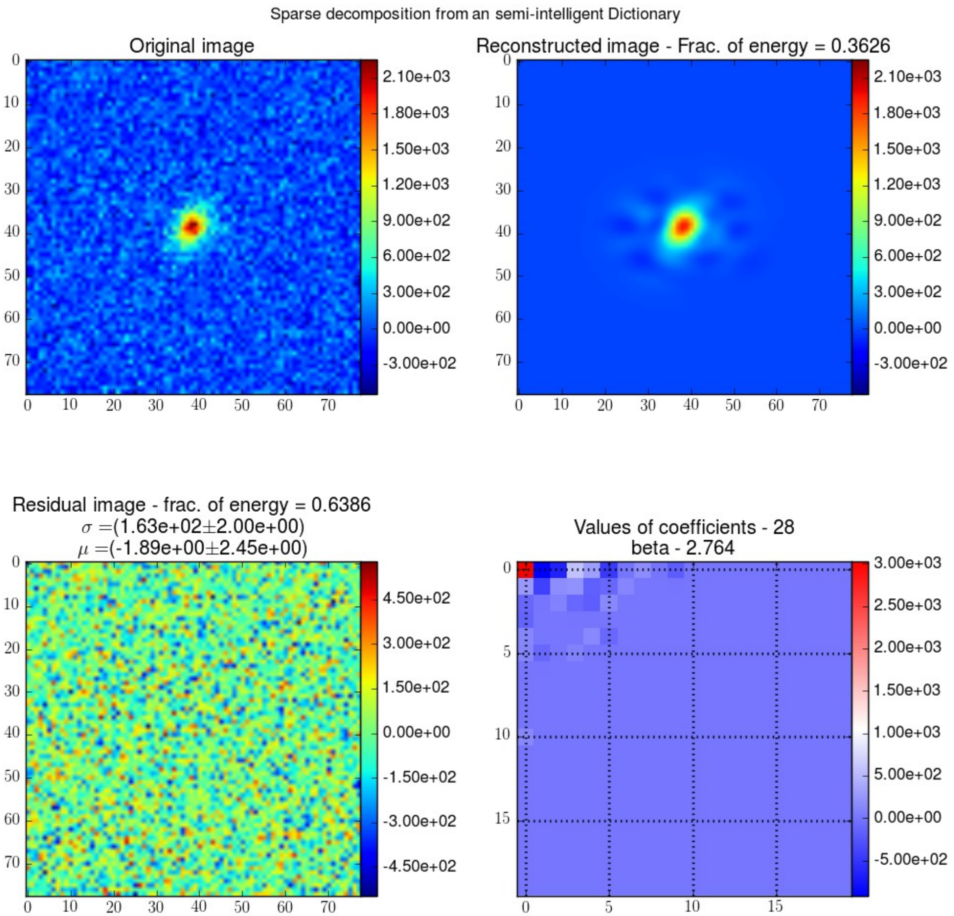


2D Cartesian shapelets

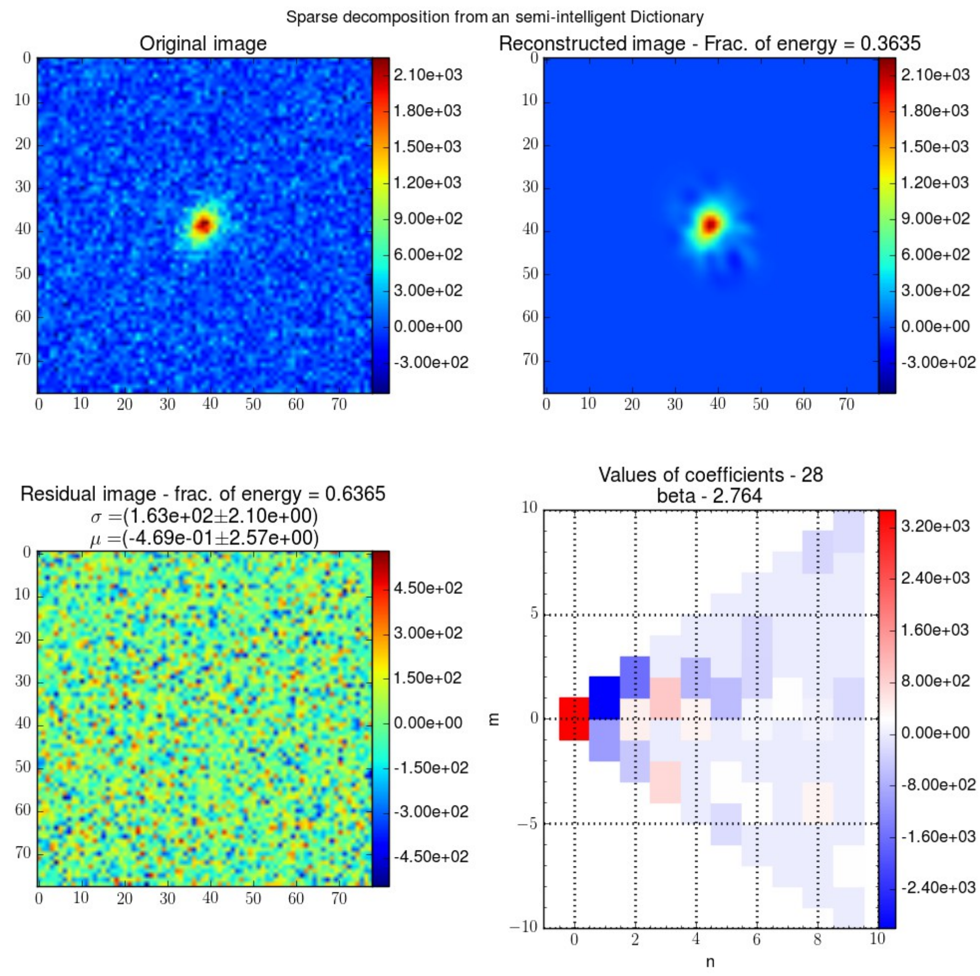


Example of 2D shapelet decomposition

28 shapelets - OMP - Cartesian



28 shapelets - OMP - Polar



Shapelets are easily transformed

$$\mathbf{x} \to \mathbf{x}' = (1 + \mathbf{\Psi})\mathbf{x} + \epsilon \qquad \mathbf{\Psi} = \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 - \rho \\ \gamma_2 + \rho & \kappa - \gamma_1 \end{pmatrix}$$

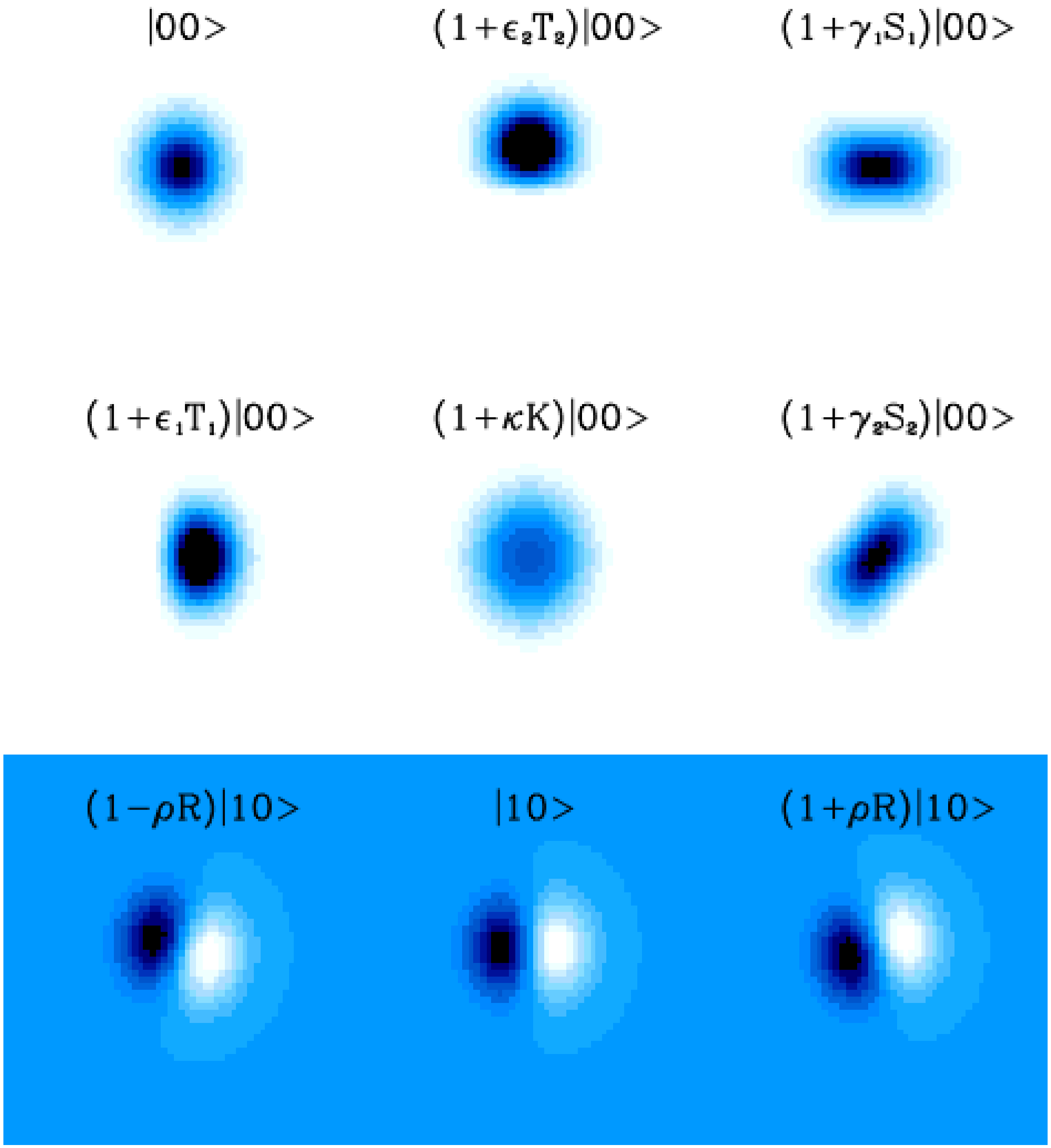
$$\downarrow \quad f(\mathbf{x}) = \sum_{i=0}^{\infty} \frac{\mathbf{x}' - \mathbf{x}}{i!} \nabla (f(\mathbf{x}))|_{\mathbf{x}=\mathbf{x}'}$$

$$\begin{aligned}
\hat{R} &= -i\left(\hat{x}_1\hat{p}_2 - \hat{x}_2\hat{p}_1\right) = \hat{a}_1\hat{a}_2^\dagger - \hat{a}_1^\dagger\hat{a}_2 \\
\hat{K} &= -i\left(\hat{x}_1\hat{p}_1 + \hat{x}_2\hat{p}_2\right) = 1 + \frac{1}{2}\left(\hat{a}_1^{\dagger 2} + \hat{a}_2^{\dagger 2} - \hat{a}_1^2 - \hat{a}_2^2\right) \\
\hat{S}_1 &= -i\left(\hat{x}_1\hat{p}_1 - \hat{x}_2\hat{p}_2\right) = \frac{1}{2}\left(\hat{a}_1^{\dagger 2} - \hat{a}_2^{\dagger 2} - \hat{a}_1^2 + \hat{a}_2^2\right) \\
\hat{S}_2 &= -i\left(\hat{x}_1\hat{p}_2 + \hat{x}_2\hat{p}_1\right) = \hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1\hat{a}_2 \\
\hat{T}_j &= -i\hat{p}_j = \frac{1}{\sqrt{2}}(\hat{a}_j^\dagger - \hat{a}_j), \quad j = 1, 2.
\end{aligned}$$

$$\downarrow$$

$$f' \simeq (1 + \rho\hat{R} + \kappa\hat{K} + \gamma_j\hat{S}_j + \epsilon_i\hat{T}_i)f$$

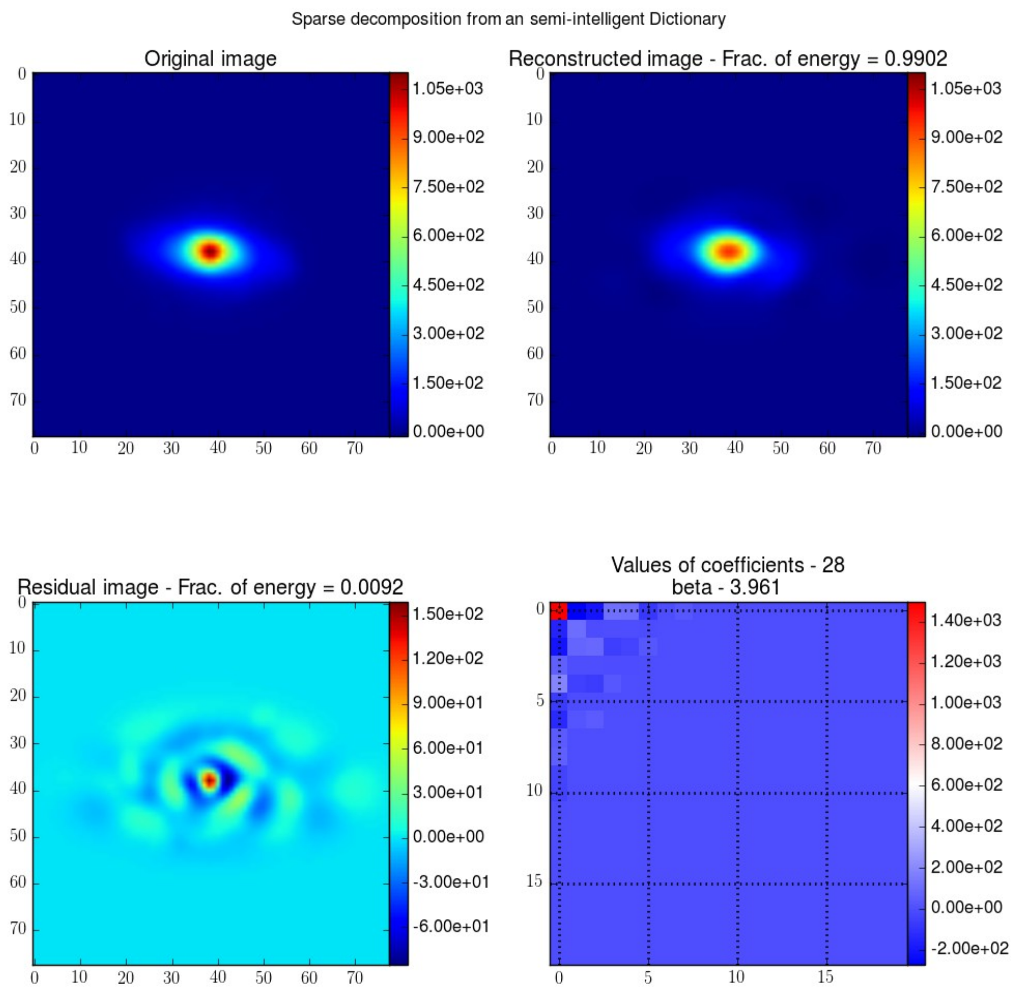
Refregier A., MNRAS, 2001



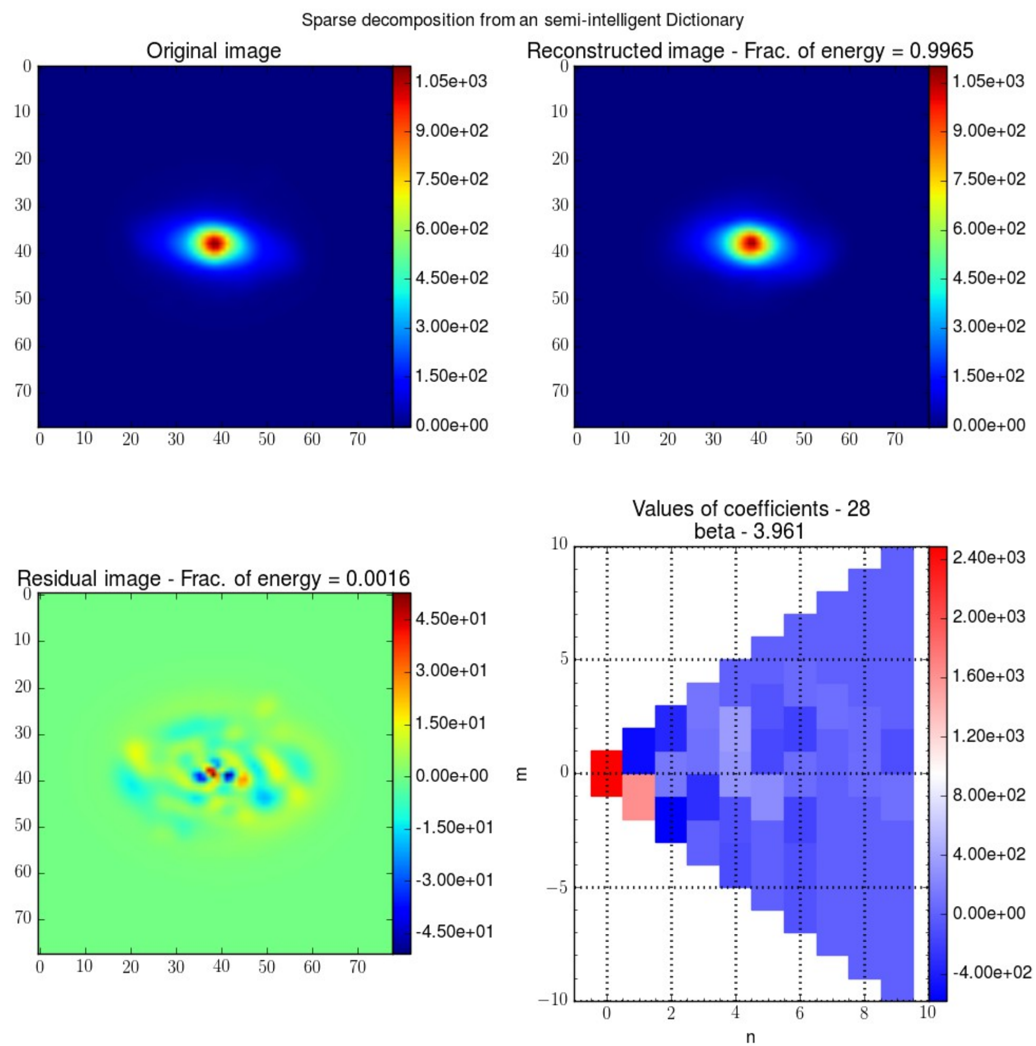
Refregier A., MNRAS, 2001

Capturing the profile well

28 shapelets - OMP - Cartesian

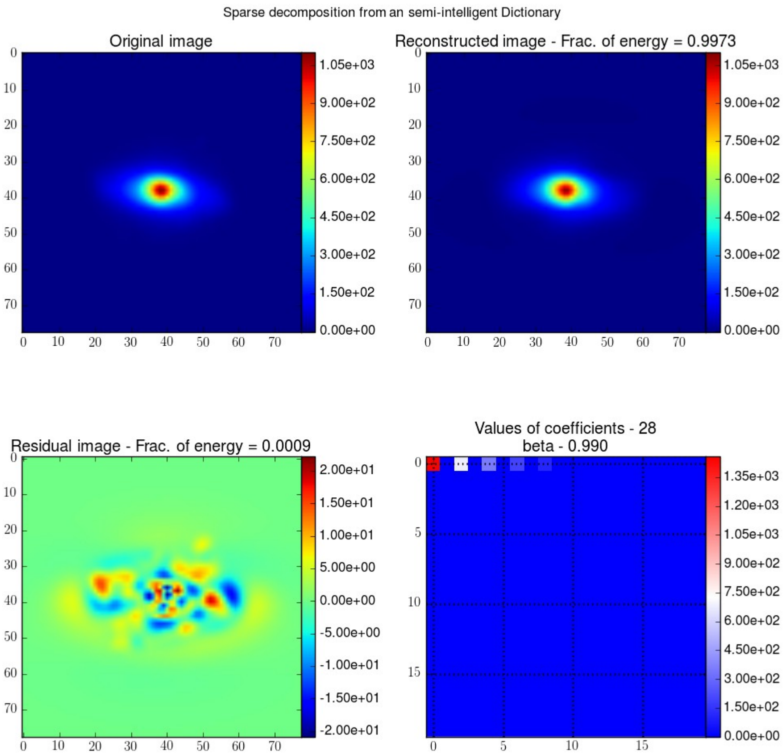


28 shapelets - OMP - Polar

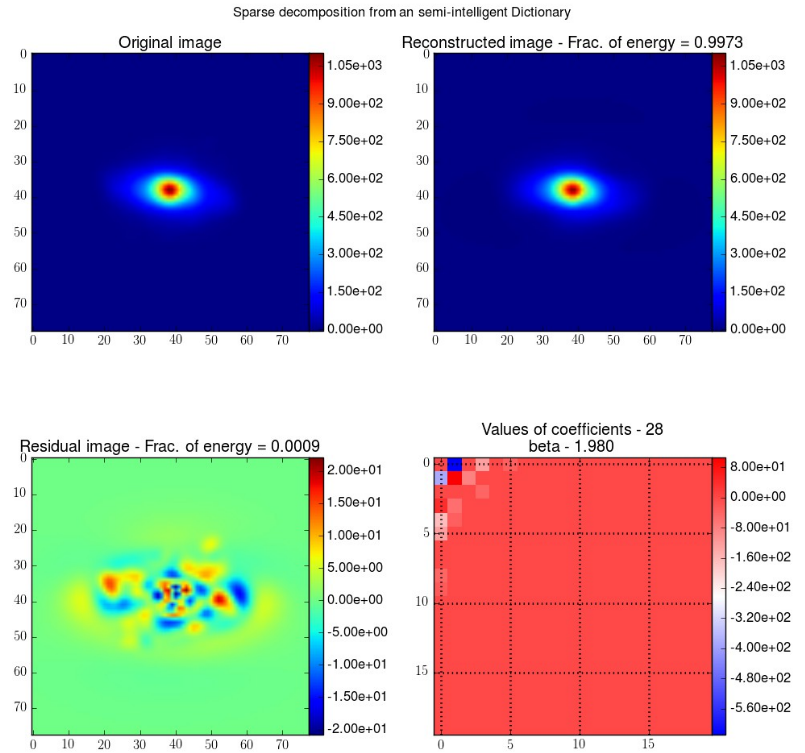


Compound XY

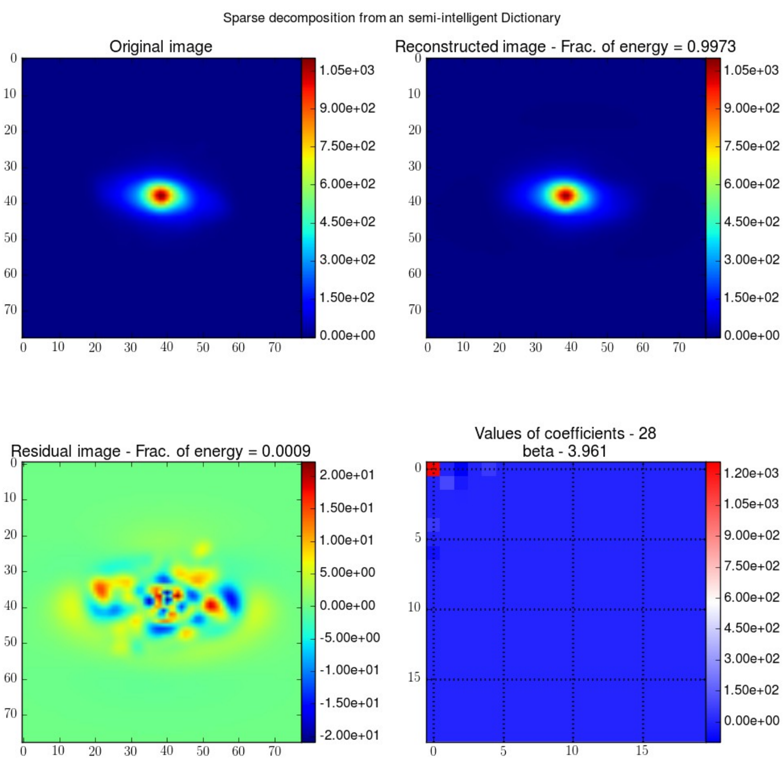
$\beta = 0.99$



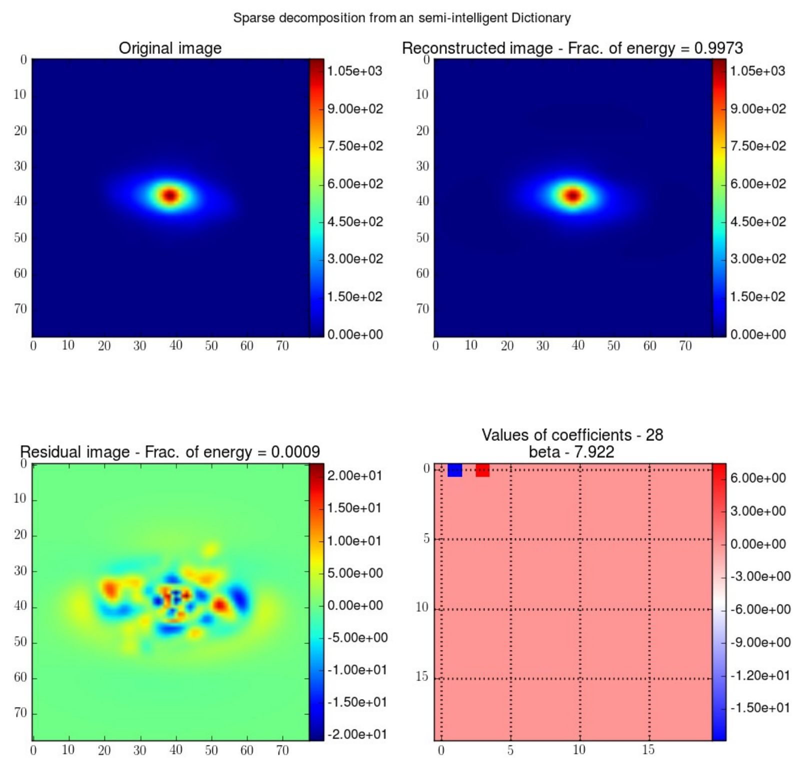
$\beta = 1.98$



$\beta = 3.97$

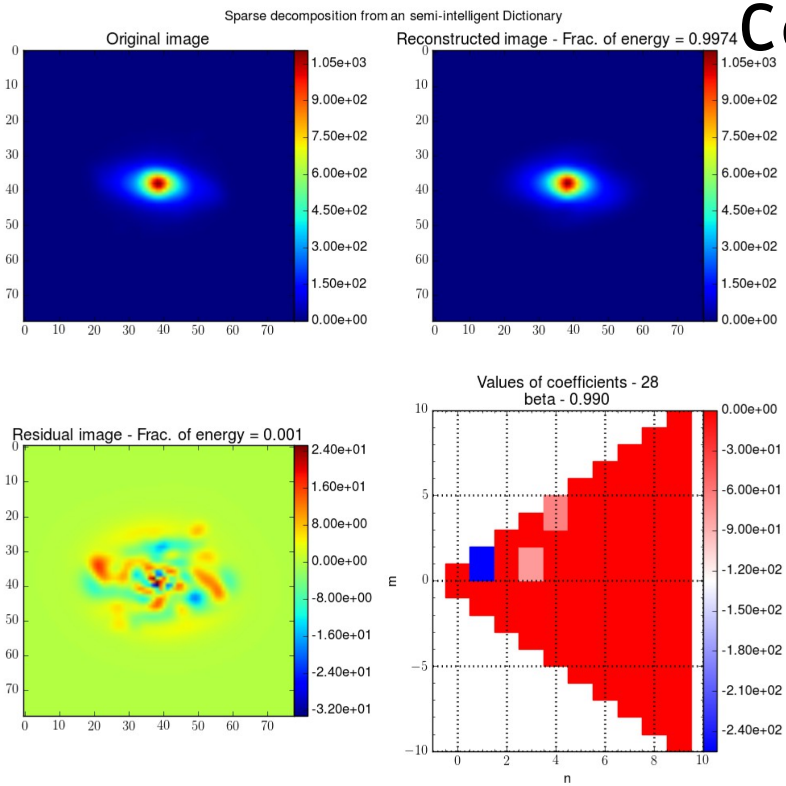


$\beta = 7.92$

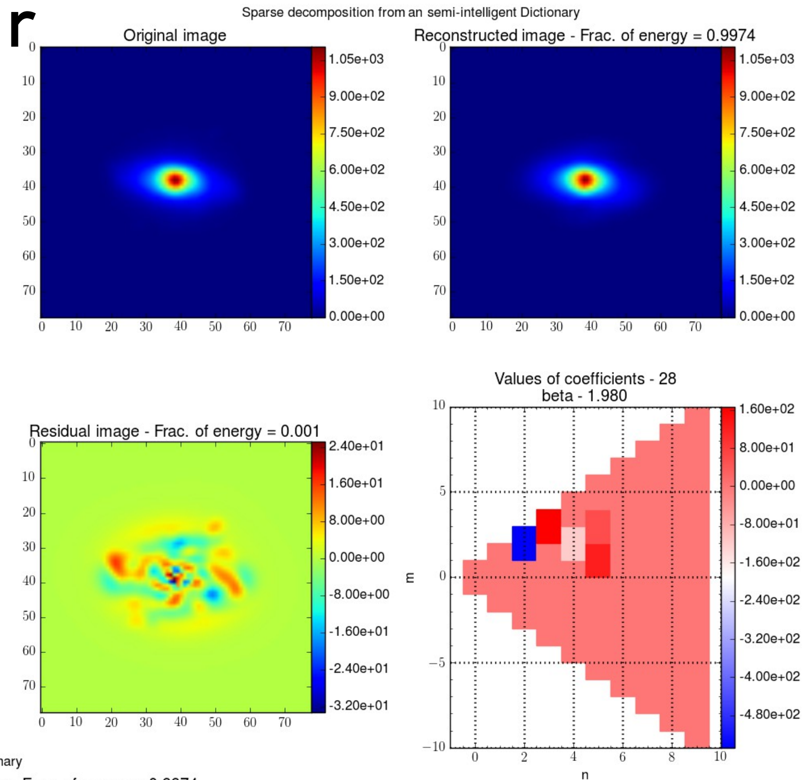


Compound Polar

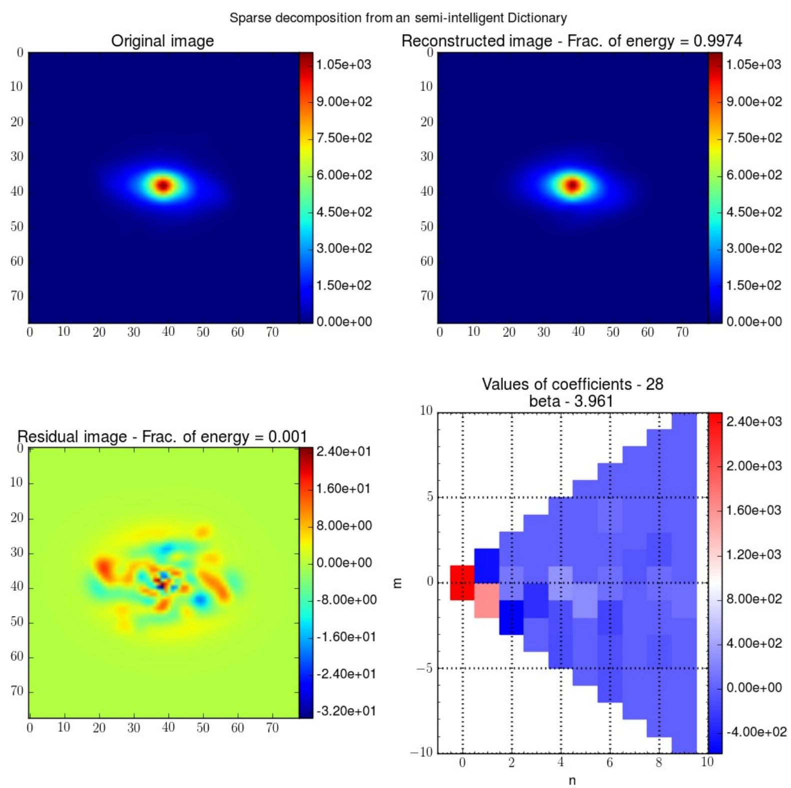
$\beta = 0.99$



$\beta = 1.98$



$\beta = 3.97$



Need to use sparse solvers

- Use of Compound basis asks for solution restriction

- Solvers considered:
  - Without free parameters:
    - Least squares

      $$\min \|y - Xw\|_2^2$$

    - Singular value decomposition (SVD)

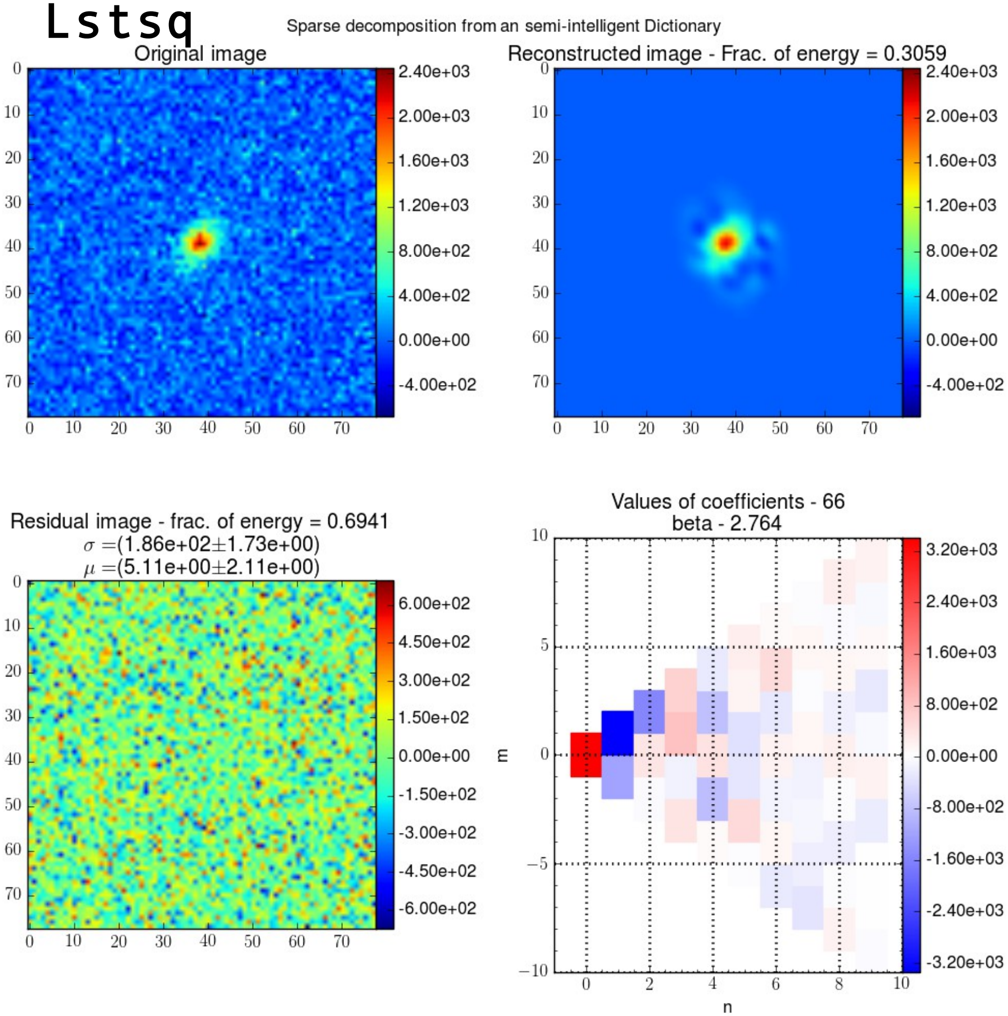
  - Sparse solvers:
    - Lasso - regression method

      $$\min \frac{1}{2N} \|y - Xw\|_2^2 + \lambda \|w\|_1$$

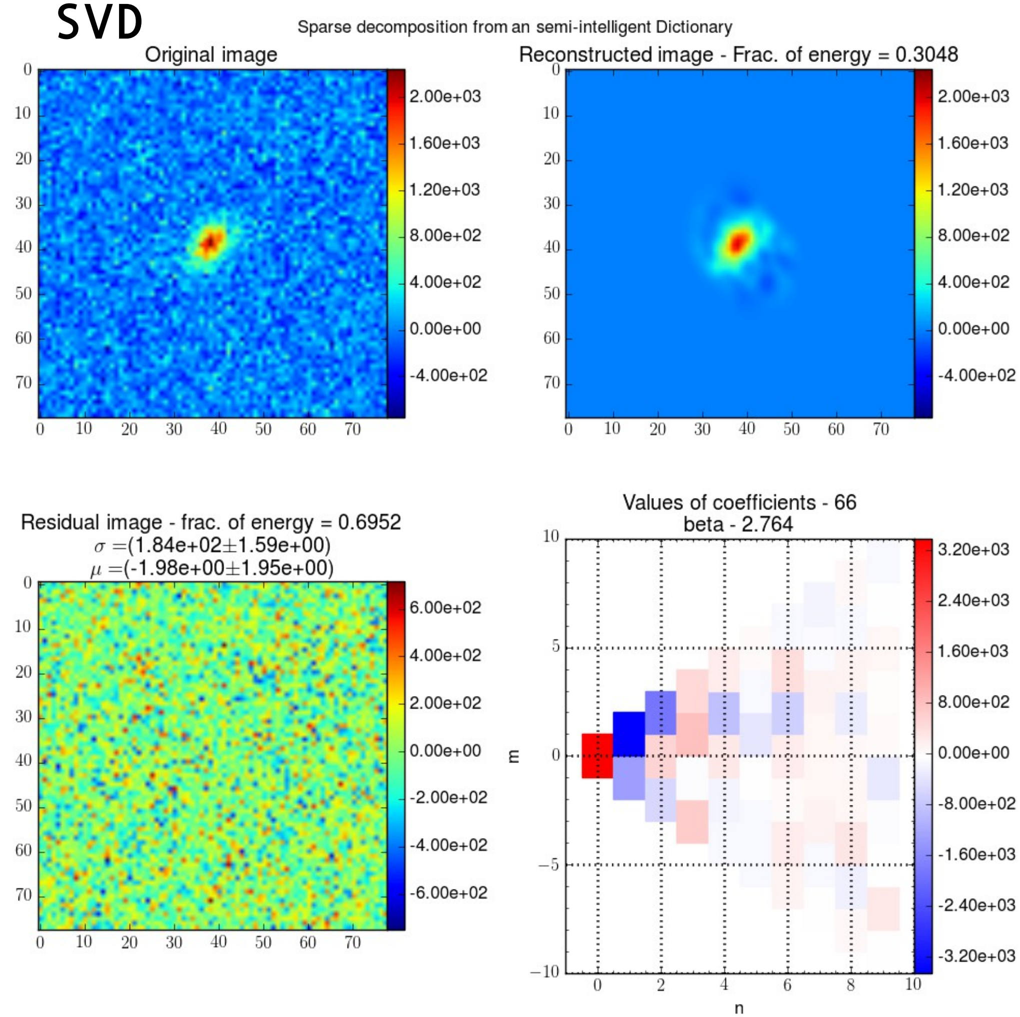
    - Orthogonal matching pursuit (OMP)

      $$\min \|y - Xw\|_2^2 \; s.t. \; \|w\|_0 \leq L$$

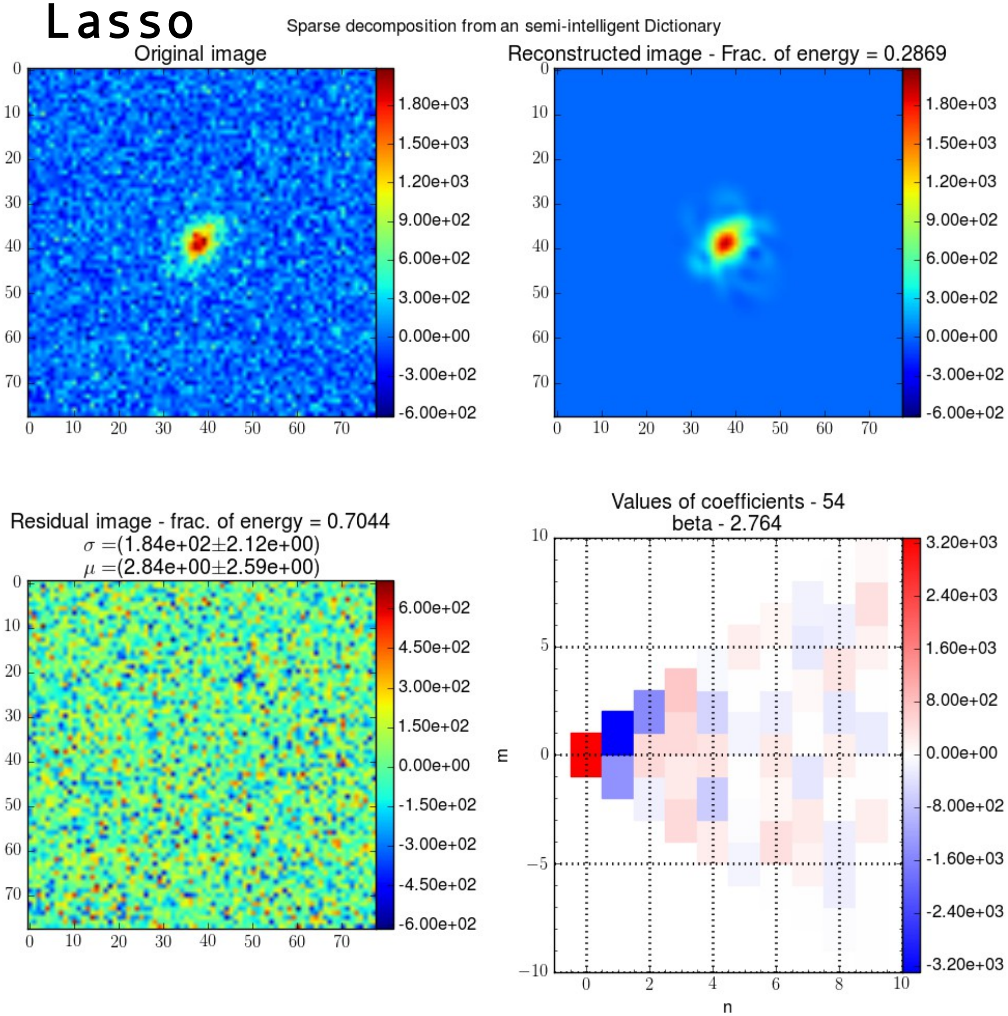
Lstsq


Sparse decomposition from an semi-intelligent Dictionary
Original image
Reconstructed image - Frac. of energy = 0.3059
Residual image - frac. of energy = 0.6941
σ =(1.86e+02±1.73e+00)
μ =(5.11e+00±2.11e+00)
Values of coefficients - 66
beta - 2.764
m
n

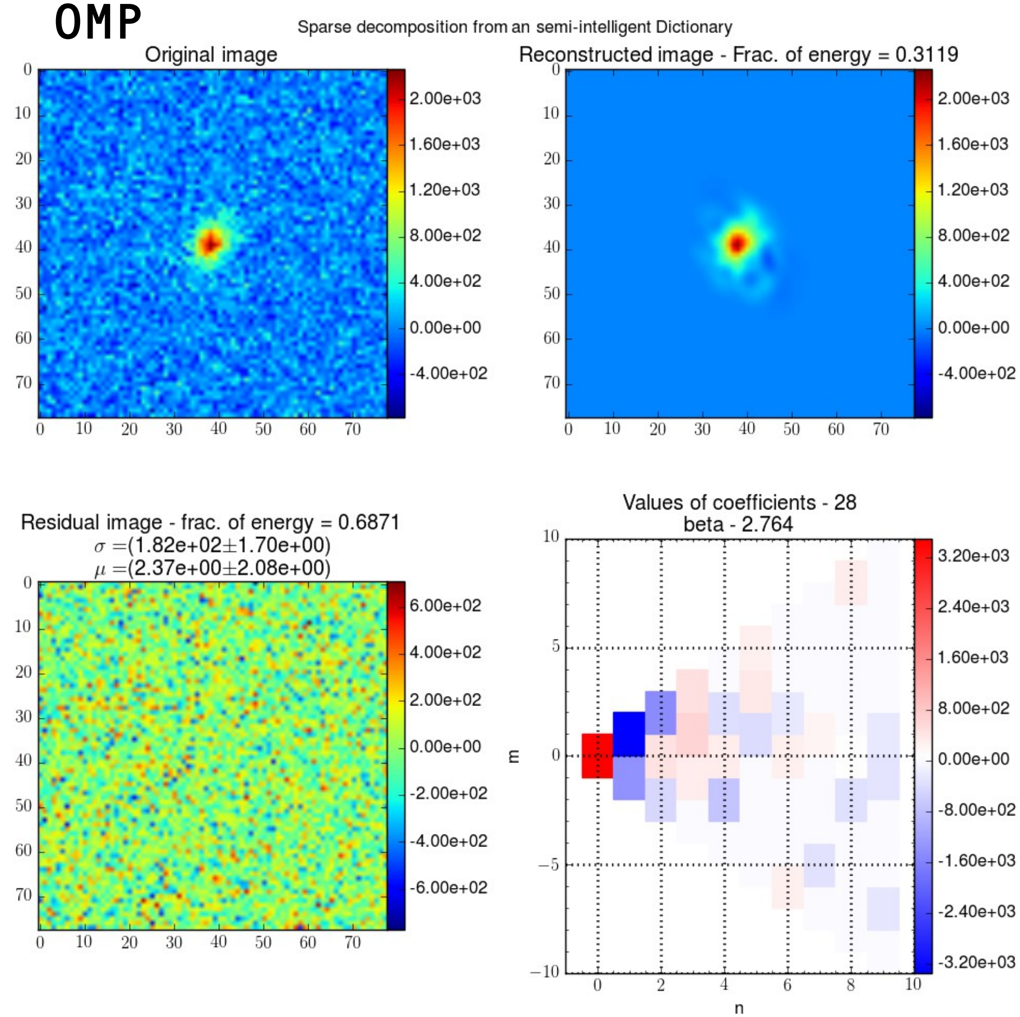
SVD


Sparse decomposition from an semi-intelligent Dictionary
Original image
Reconstructed image - Frac. of energy = 0.3048
Residual image - frac. of energy = 0.6952
σ =(1.84e+02±1.59e+00)
μ =(-1.98e+00±1.95e+00)
Values of coefficients - 66
beta - 2.764
m
n

Lasso

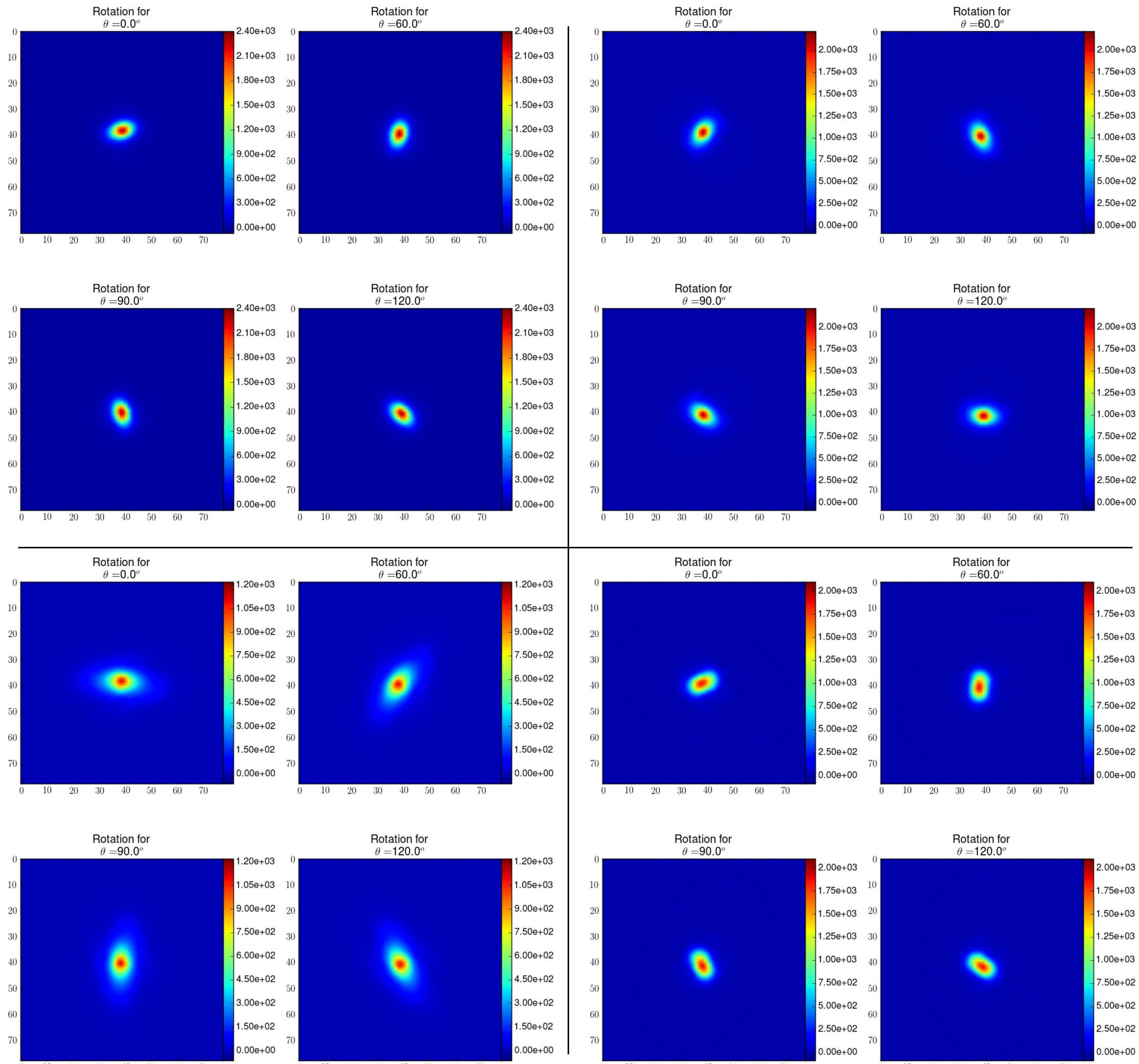

Sparse decomposition from an semi-intelligent Dictionary
Original image
Reconstructed image - Frac. of energy = 0.2869
Residual image - frac. of energy = 0.7044
σ =(1.84e+02±2.12e+00)
μ =(2.84e+00±2.59e+00)
Values of coefficients - 54
beta - 2.764
m
n

OMP

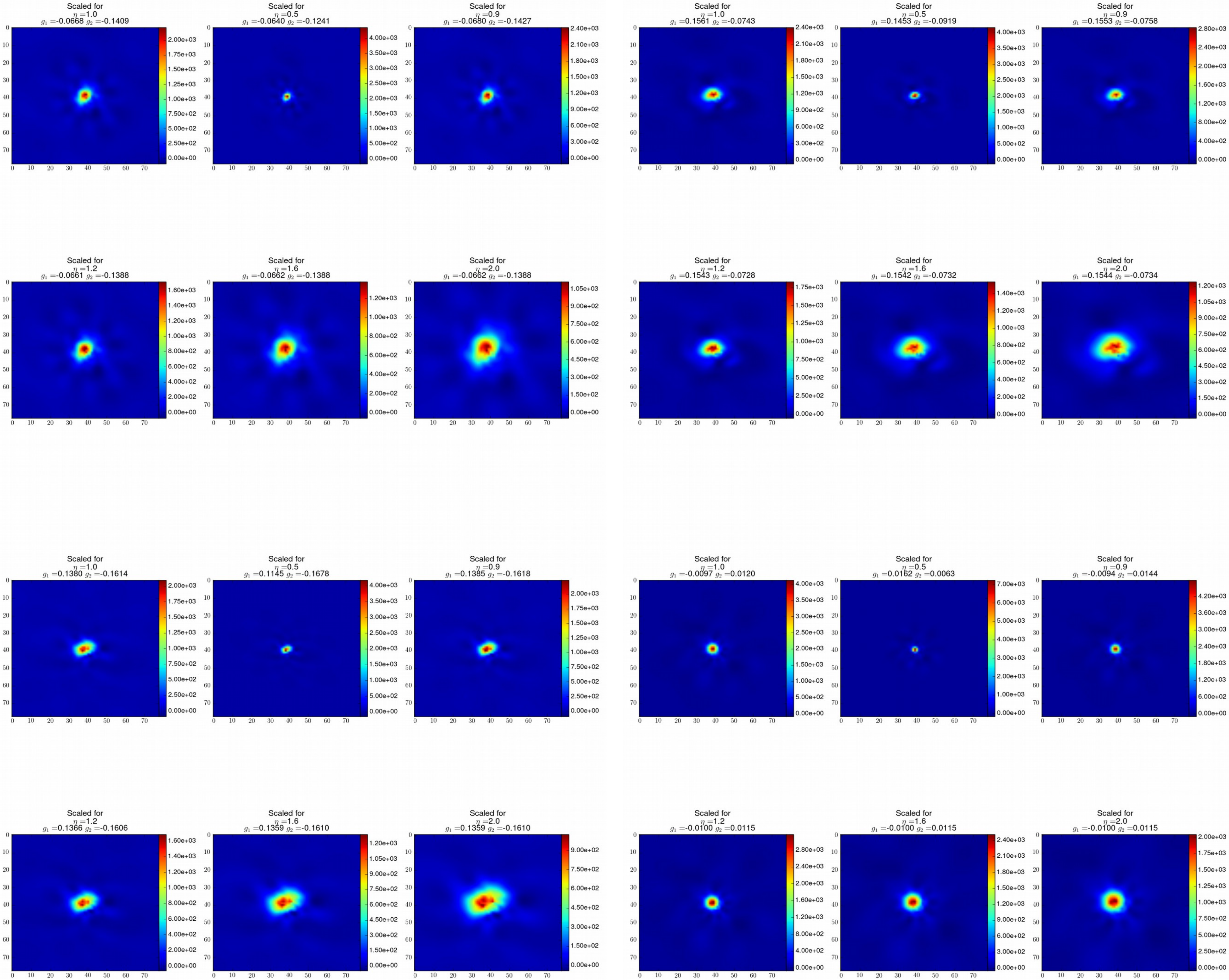

Sparse decomposition from an semi-intelligent Dictionary
Original image
Reconstructed image - Frac. of energy = 0.3119
Residual image - frac. of energy = 0.6871
σ =(1.82e+02±1.70e+00)
μ =(2.37e+00±2.08e+00)
Values of coefficients - 28
beta - 2.764
m
n

Creating mock galaxies

Rotation

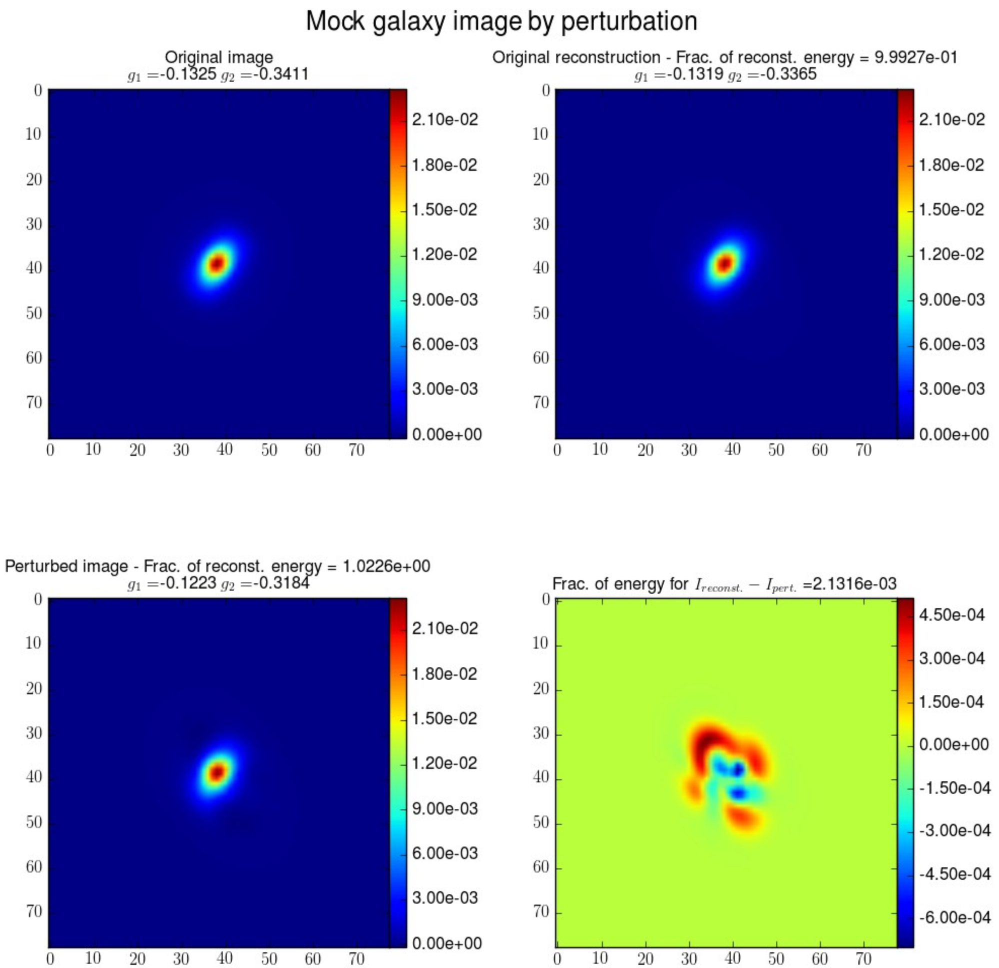


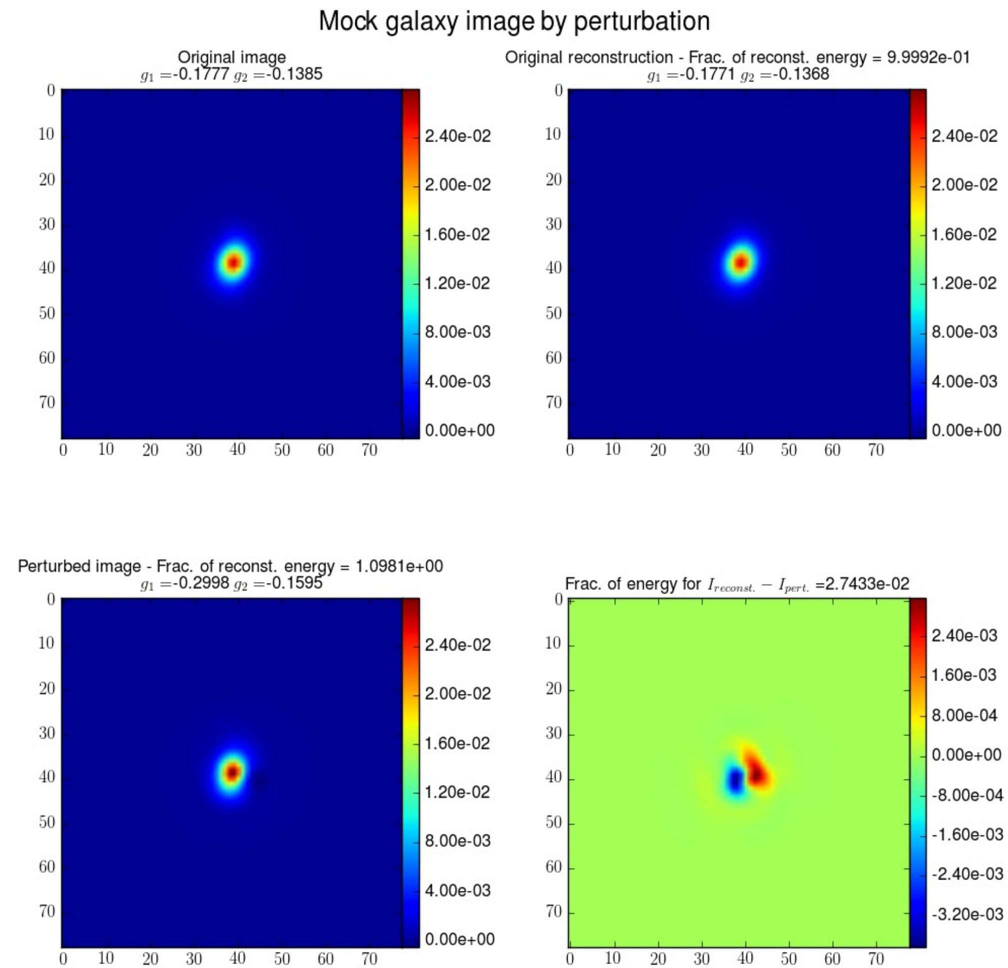
Scaling



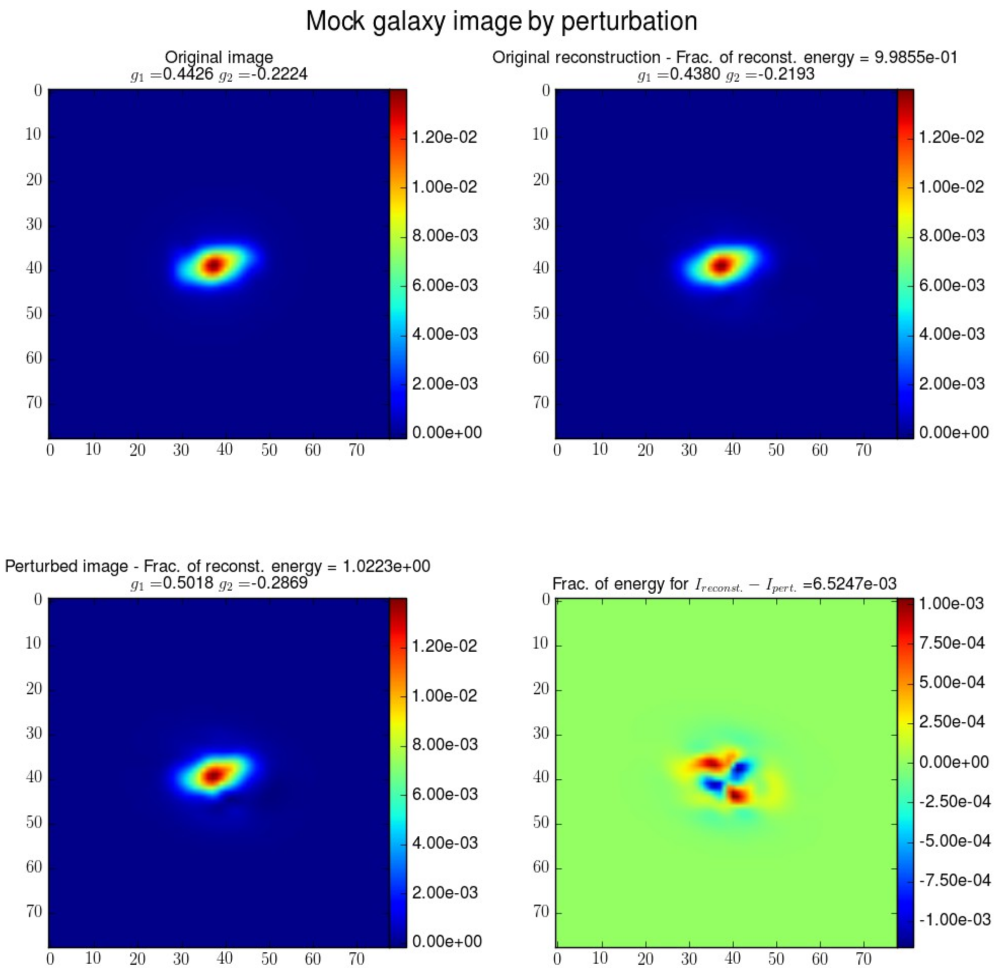
Scaled for η=1.0 g_1=-0.0668 g_2=-0.1409
Scaled for η=0.5 g_1=-0.0640 g_2=-0.1241
Scaled for η=0.9 g_1=-0.0680 g_2=-0.1427
Scaled for η=1.0 g_1=0.1561 g_2=-0.0743
Scaled for η=0.5 g_1=0.1453 g_2=-0.0919
Scaled for η=0.9 g_1=0.1553 g_2=-0.0758
Scaled for η=1.2 g_1=-0.0661 g_2=-0.1388
Scaled for η=1.6 g_1=-0.0662 g_2=-0.1388
Scaled for η=2.0 g_1=-0.0662 g_2=-0.1388
Scaled for η=1.2 g_1=0.1543 g_2=-0.0728
Scaled for η=1.6 g_1=0.1542 g_2=-0.0732
Scaled for η=2.0 g_1=0.1544 g_2=-0.0734
Scaled for η=1.0 g_1=0.1380 g_2=-0.1614
Scaled for η=0.5 g_1=0.1145 g_2=-0.1678
Scaled for η=0.9 g_1=0.1385 g_2=-0.1618
Scaled for η=1.0 g_1=-0.0097 g_2=0.0120
Scaled for η=0.5 g_1=0.0162 g_2=0.0063
Scaled for η=0.9 g_1=-0.0094 g_2=0.0144
Scaled for η=1.2 g_1=0.1366 g_2=-0.1606
Scaled for η=1.6 g_1=0.1359 g_2=-0.1610
Scaled for η=2.0 g_1=0.1359 g_2=-0.1610
Scaled for η=1.2 g_1=-0.0100 g_2=0.0115
Scaled for η=1.6 g_1=-0.0100 g_2=0.0115
Scaled for η=2.0 g_1=-0.0100 g_2=0.0115

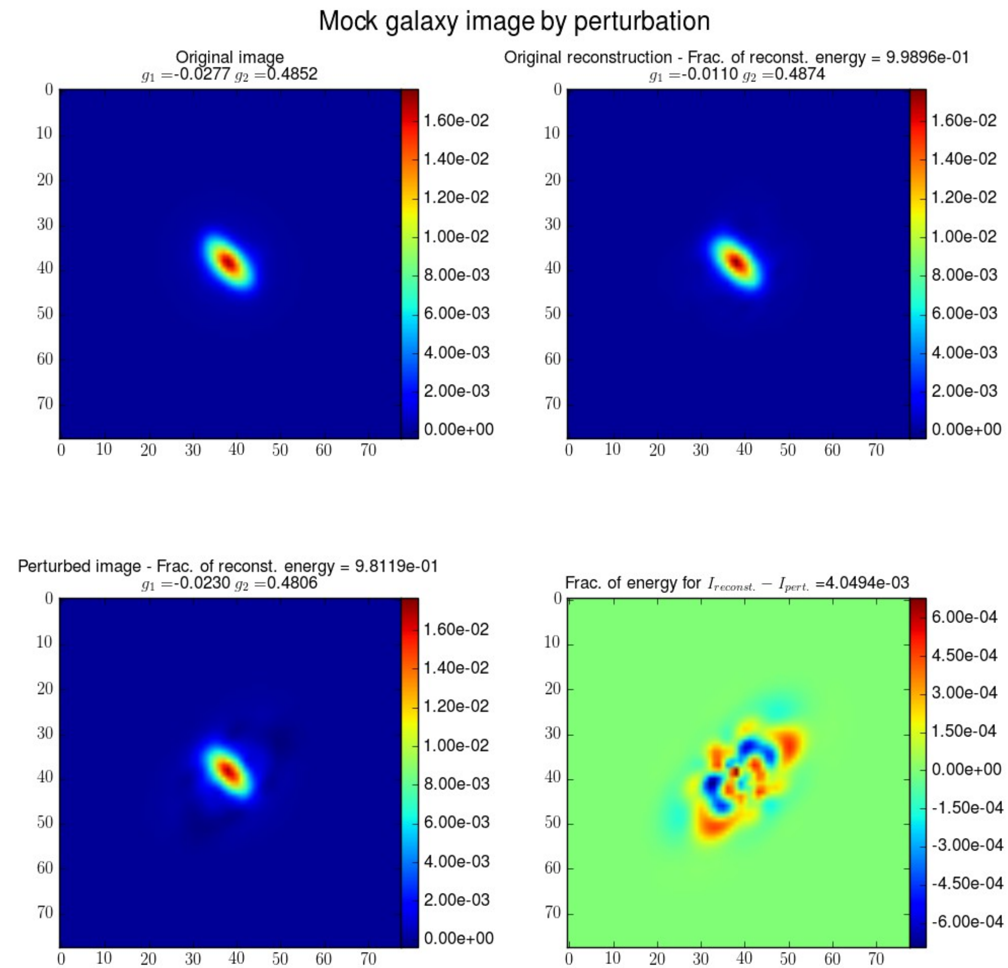
Perturbing the coefficients





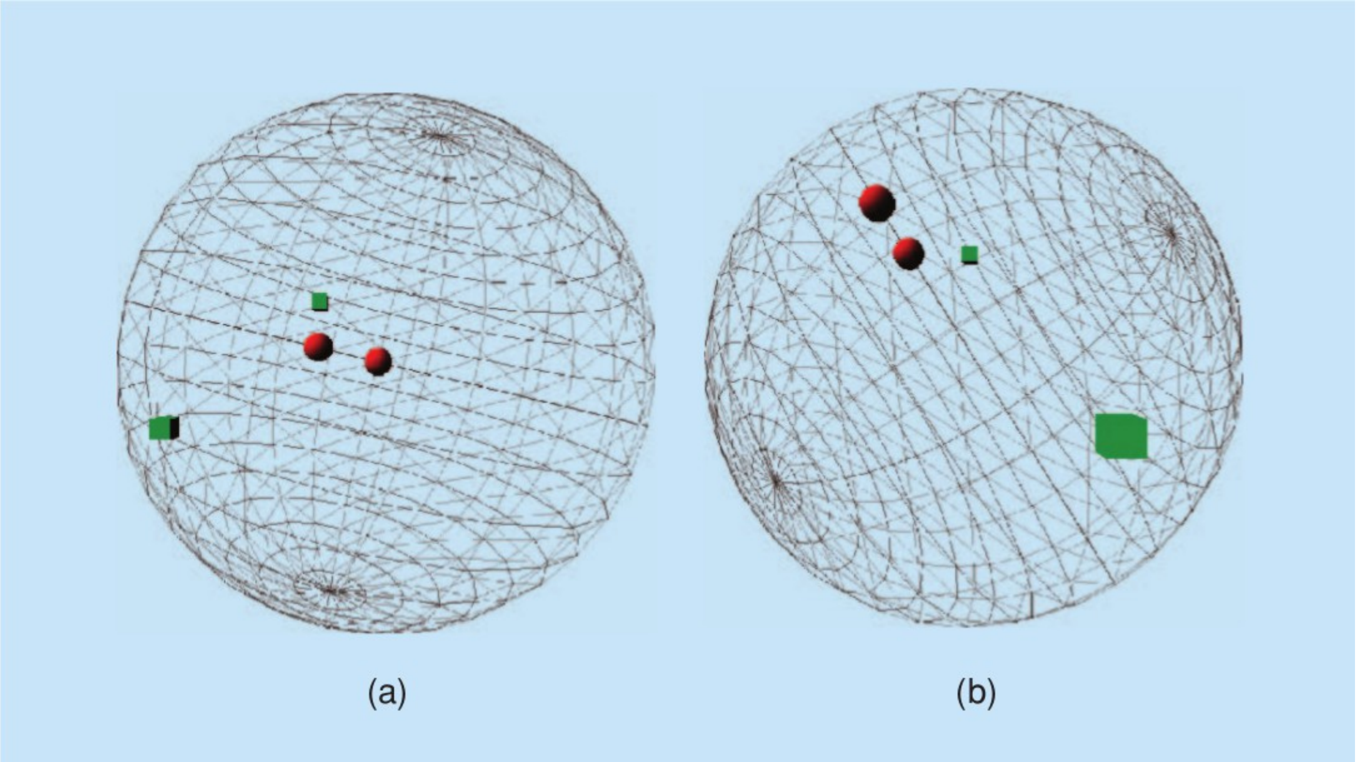
Perturbing the coefficients



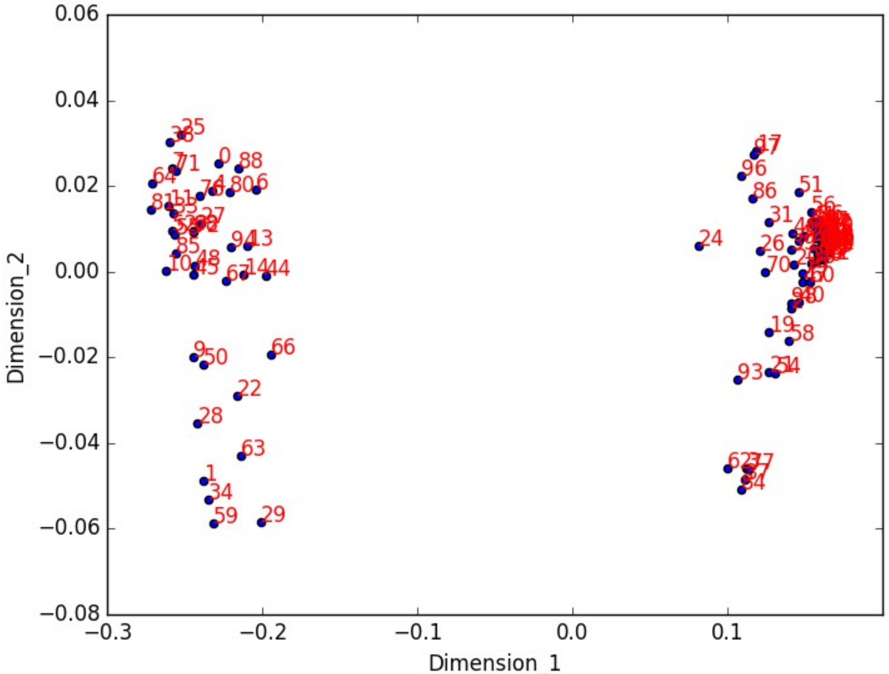


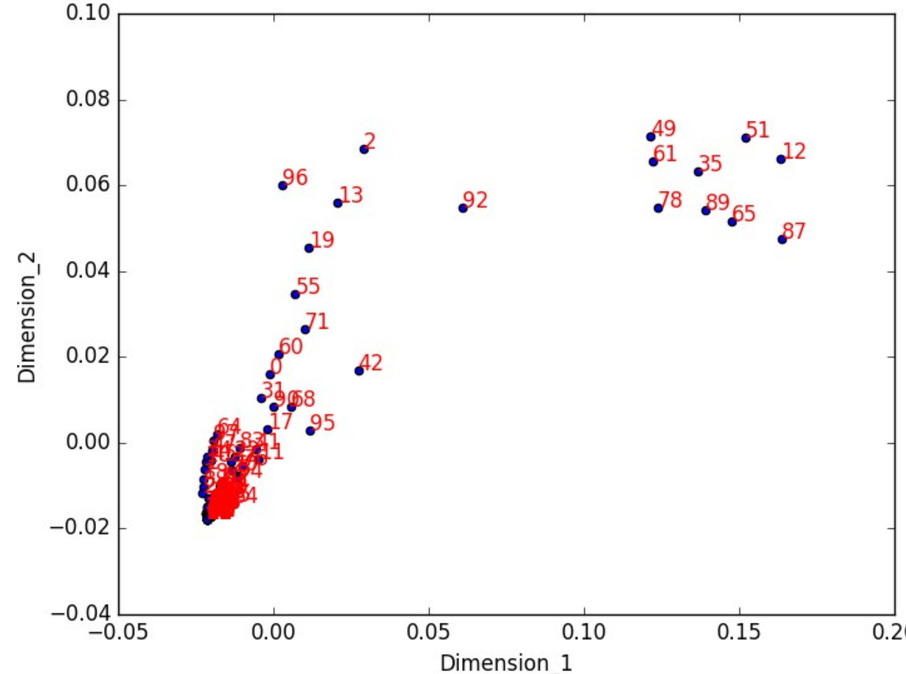
Motivation for clustering

- We want to do re-sampling of the high dimensional distribution
- Generally one needs to project down the problem

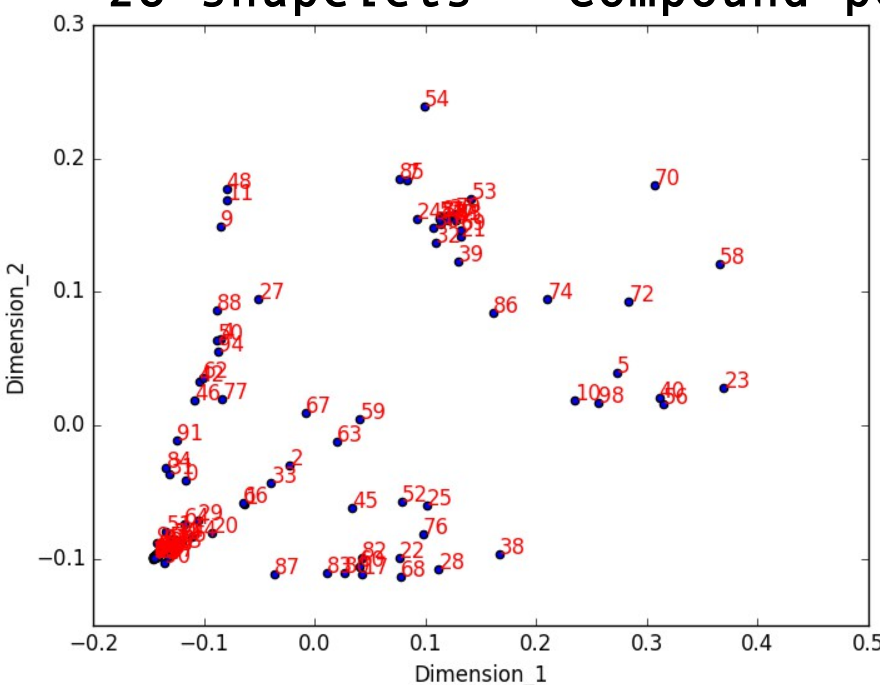


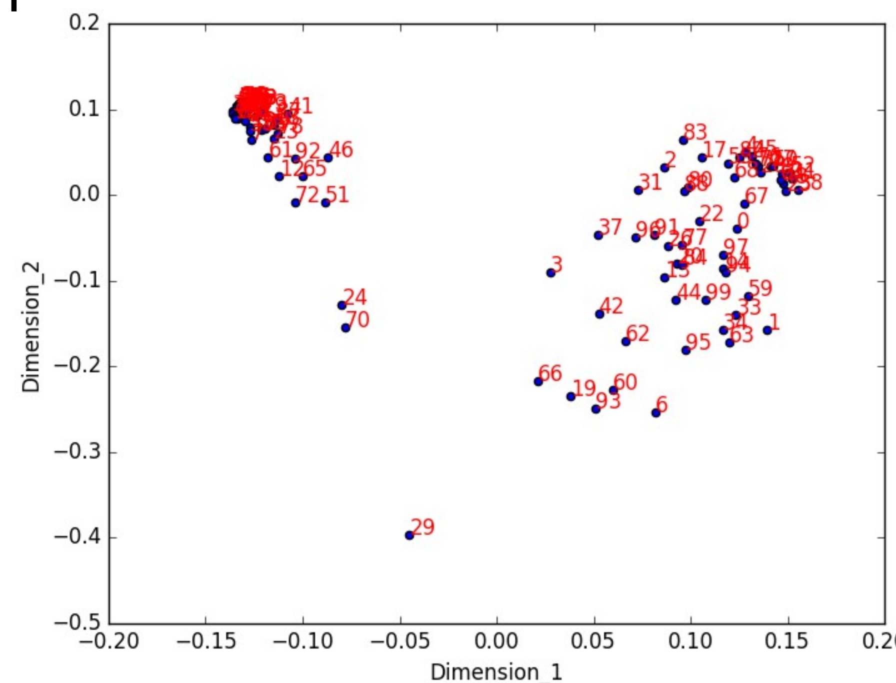
28 shapelets - Compound XY



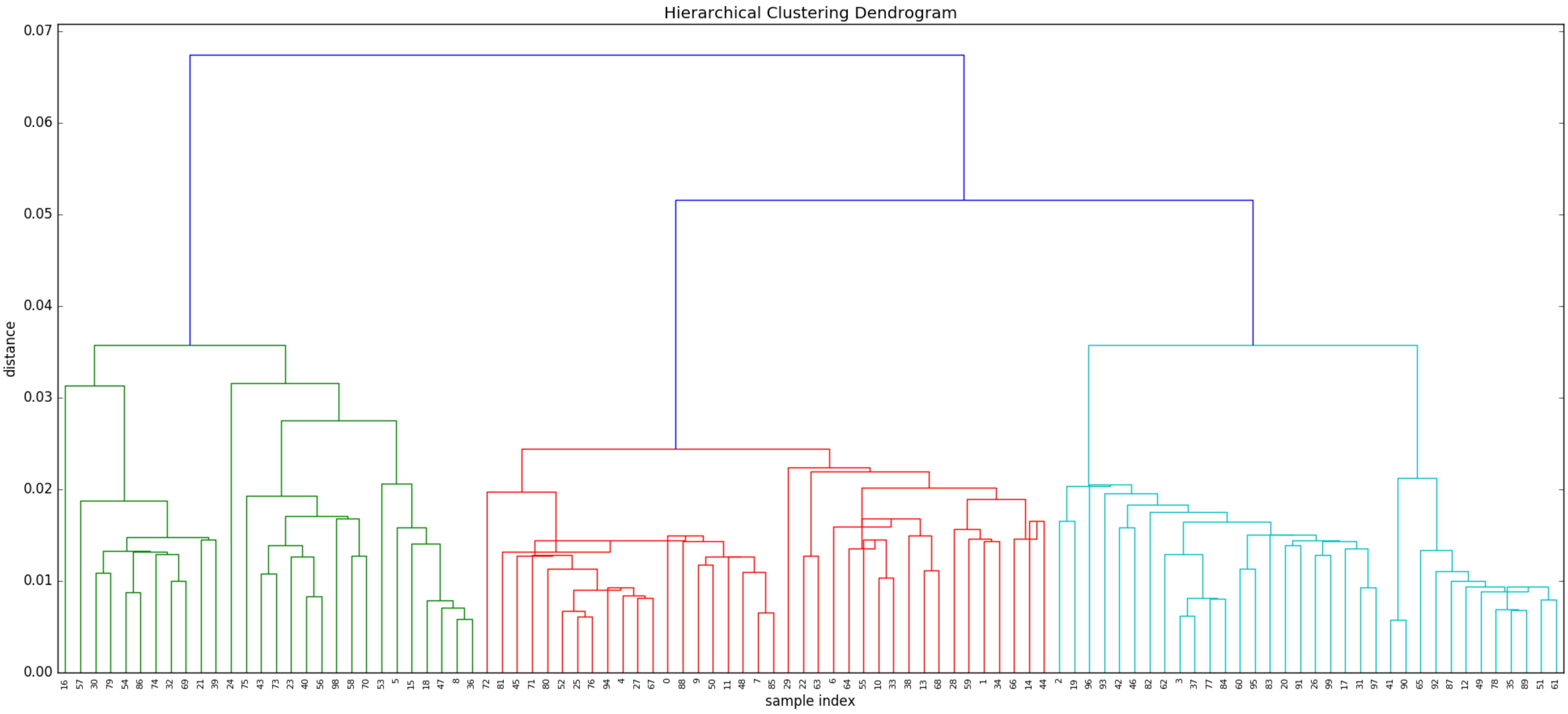


28 shapelets - Compound polar



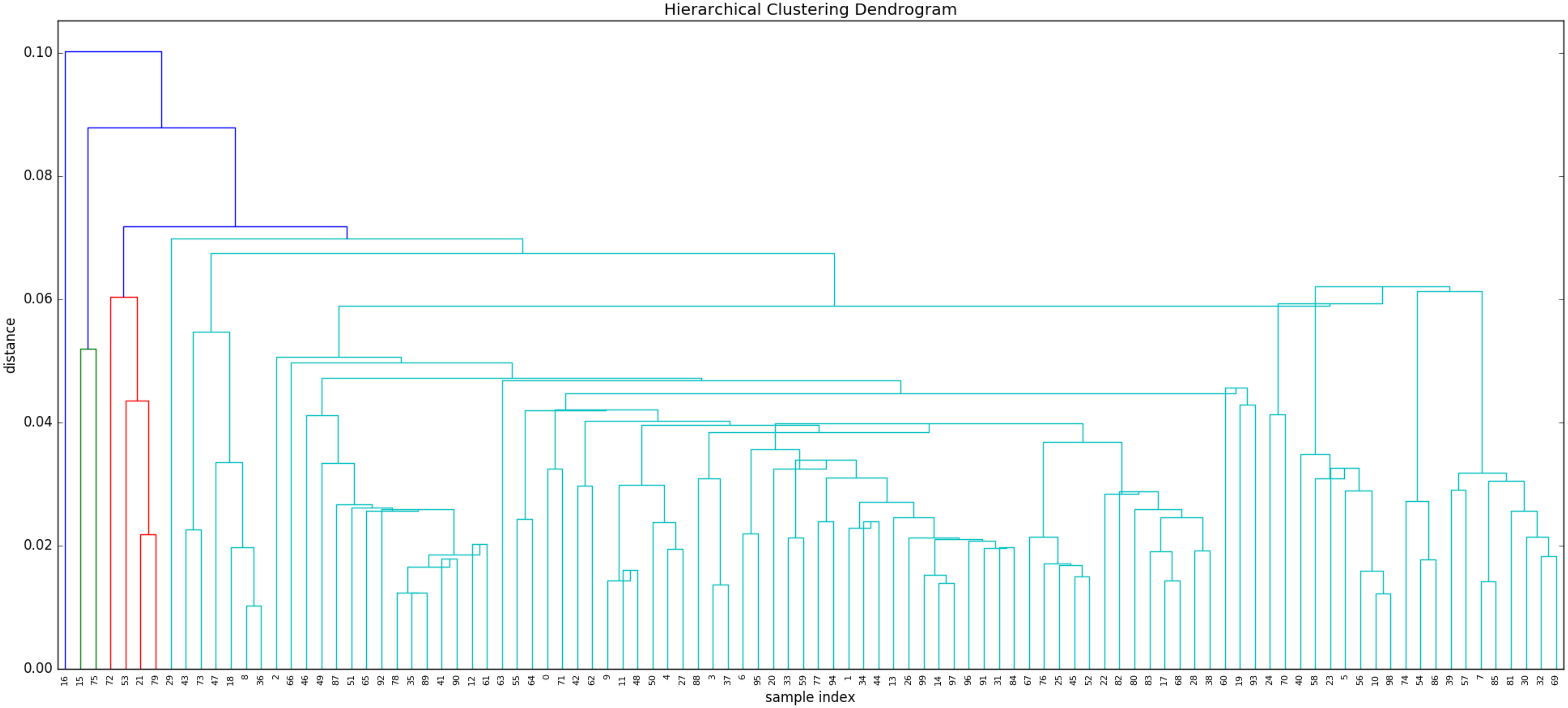


Finding clustering



28 shapelets - Compound XY

Finding clustering



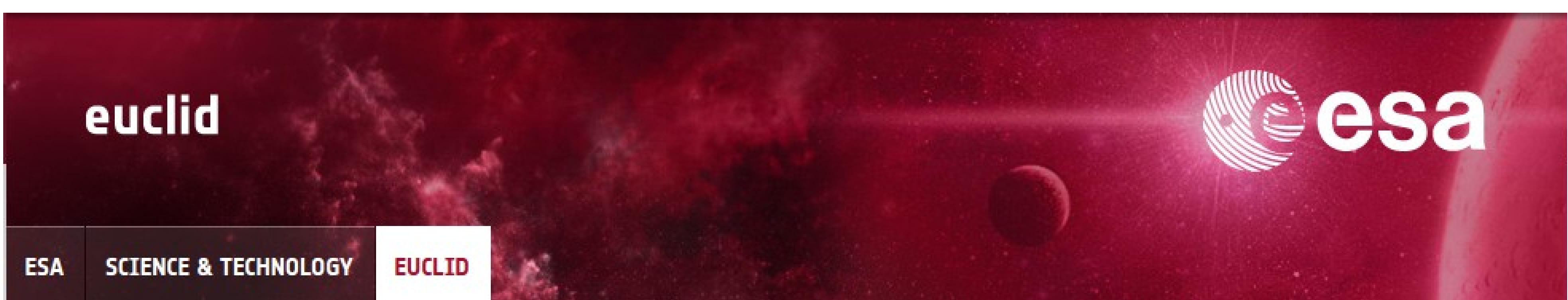
28 shapelets - Compound Polar

Conclusion and Future work

- Shapelets are good at capturing the profile and easily manipulated
- Potential of creating good mock catalogues with high enough details for a good bias estimate
  (Self organizing maps)
- Need to asses the problem of PSF deconvolution

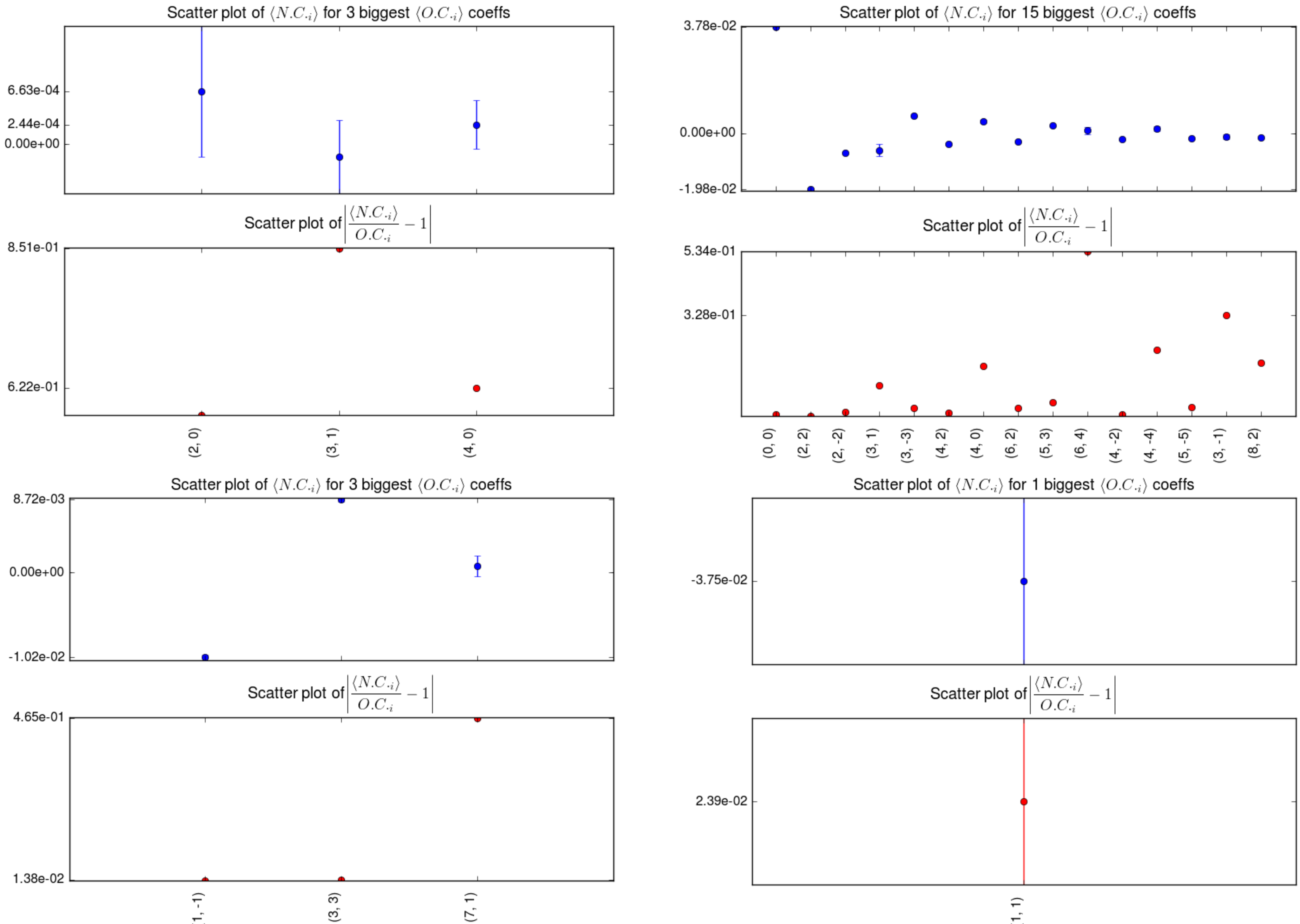
- Especially useful for the upcoming missions → Euclid mission

- It would be good to see how well the bias can be reduced with shapelets

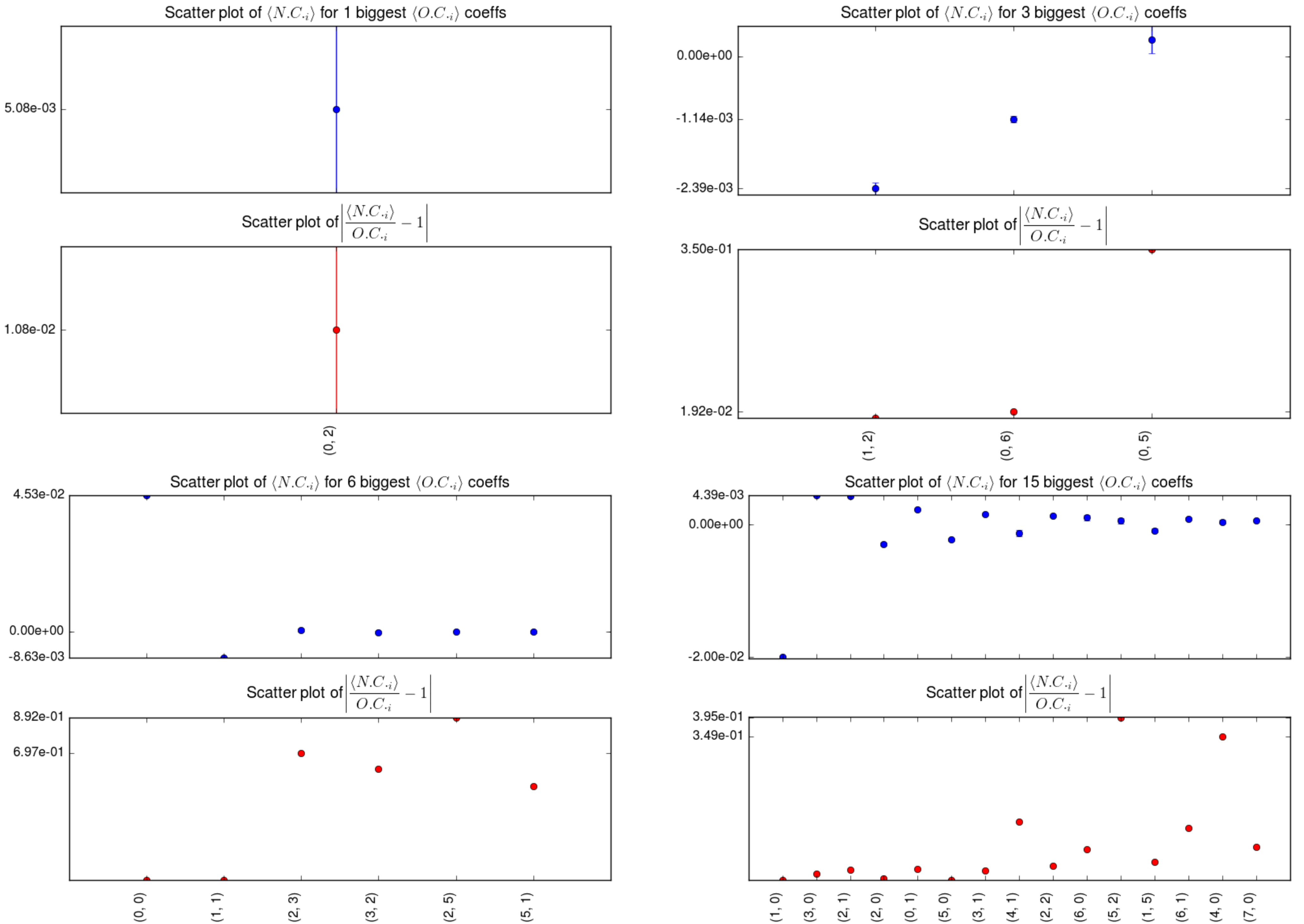


Thank you for your attention.

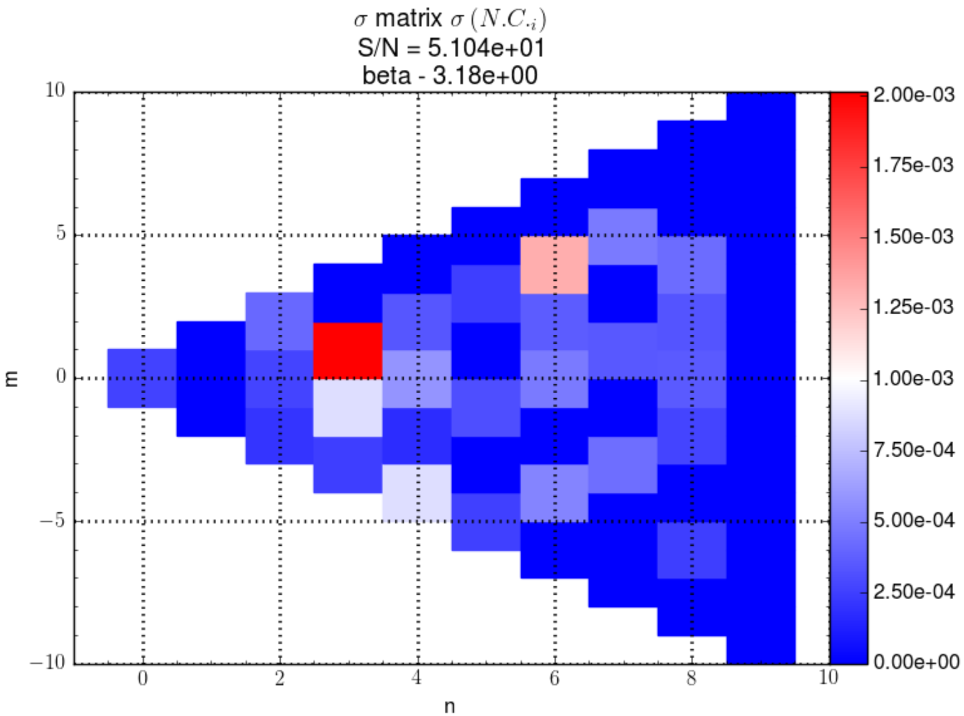
Stability tests

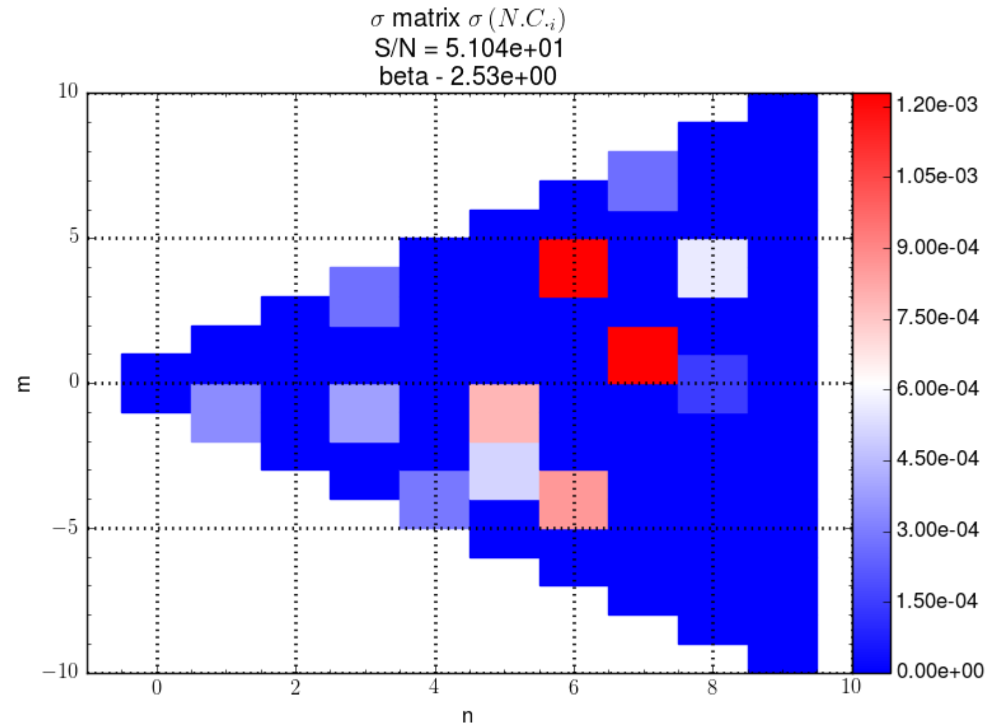


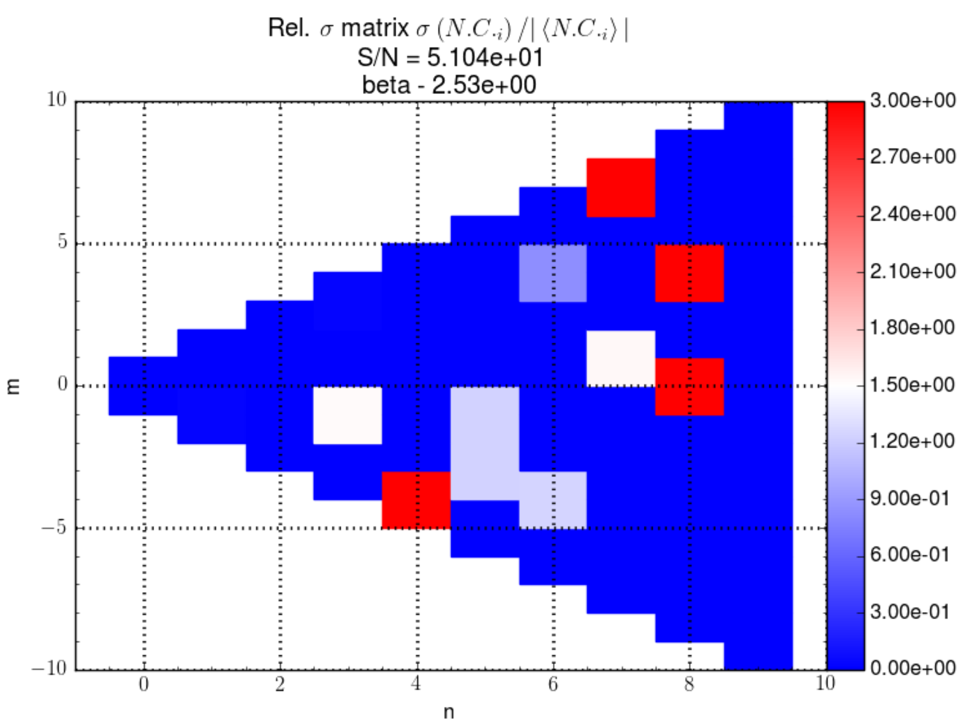
Stability tests

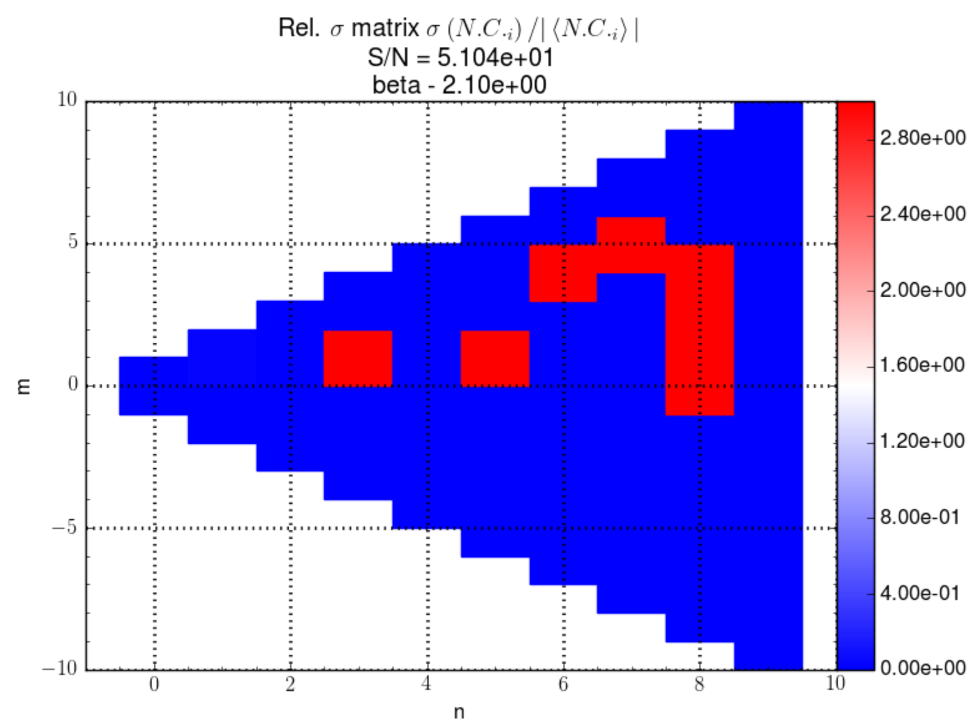


Stability tests

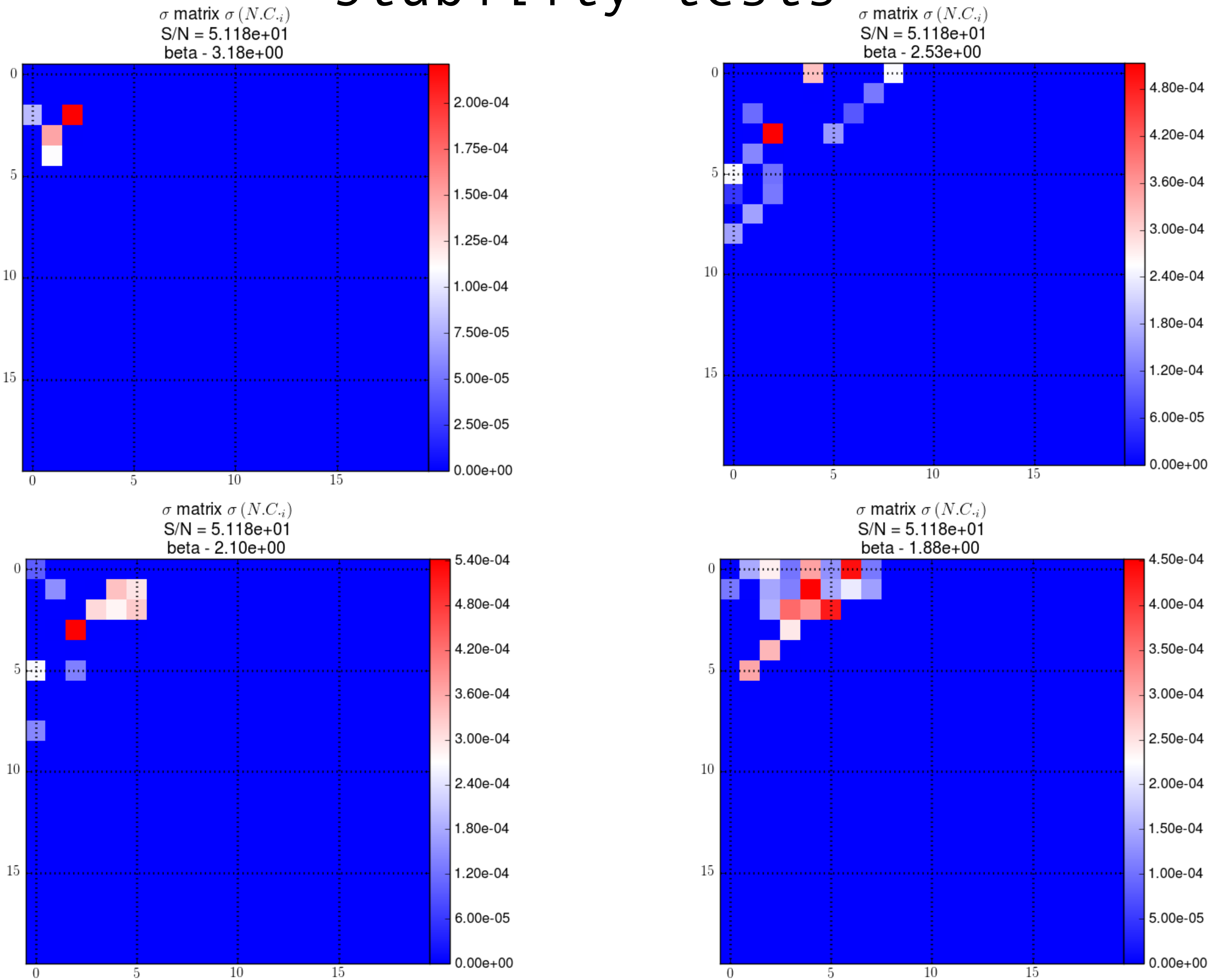








Stability tests



A little bit more on the algorithms used

Task: Approximate the solution of (P_0): $\min_{\mathbf{x}} \ \|\mathbf{x}\|_0$ subject to $\mathbf{Ax} = \mathbf{b}$.
Parameters: We are given the matrix $\mathbf{A}$, the vector $\mathbf{b}$, and the error threshold ϵ_0.
Initialization: Initialize $k = 0$, and set

- The initial solution $\mathbf{x}^0 = \mathbf{0}$.
- The initial residual $\mathbf{r}^0 = \mathbf{b} - \mathbf{Ax}^0 = \mathbf{b}$.
- The initial solution support $\mathcal{S}^0 = Support\{\mathbf{x}^0\} = \emptyset$.

Main Iteration: Increment k by 1 and perform the following steps:

- **Sweep:** Compute the errors $\epsilon(j) = \min_{z_j} \|\mathbf{a}_j z_j - \mathbf{r}^{k-1}\|_2^2$ for all j using the optimal choice $z_j^* = \mathbf{a}_j^T \mathbf{r}^{k-1} / \|\mathbf{a}_j\|_2^2$.
- **Update Support:** Find a minimizer, j_0 of $\epsilon(j)$: $\forall\ j \notin \mathcal{S}^{k-1}$, $\epsilon(j_0) \le \epsilon(j)$, and update $\mathcal{S}^k = \mathcal{S}^{k-1} \cup \{j_0\}$.
- **Update Provisional Solution:** Compute $\mathbf{x}^k$, the minimizer of $\|\mathbf{Ax} - \mathbf{b}\|_2^2$ subject to $Support\{\mathbf{x}\} = \mathcal{S}^k$.
- **Update Residual:** Compute $\mathbf{r}^k = \mathbf{b} - \mathbf{Ax}^k$.
- **Stopping Rule:** If $\|\mathbf{r}^k\|_2 < \epsilon_0$, stop. Otherwise, apply another iteration.

Output: The proposed solution is $\mathbf{x}^k$ obtained after k iterations.

A little bit more on the algorithms used