Optically Induced Coherence Effects in Quantum Metamaterials
Self-induced Transparency and Dicke Superradiance

Nikos Lazarides[1], George Tsironis[1,3] and Zoran Ivić[1,2,3].

[1] CQCN, Department of Physics, University of Crete,Heraklion, Greece
[2]"Vinča" Institute of Nuclear Sciences, University of Belgrade, Serbia
[3]National University of Science and Technology MISiS, Moscow, Russia

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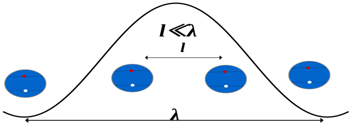
Outline

- Abstract
- Introduction
- SIT in QMM
- Conclusion

Overview of research

- Propagation of the Electromagnetic radiation in quantum metamatematirials (QMM)
  - QMM $\Leftrightarrow$ one-dimensional chain of the large number of aligned identical superconducting charge qubit (SCCQB)
    - Rakhmanov, Zagoskin, Savel'ev, Nori, PRB **77** 144507 (2008)
- Theoretical prediction of two remarkable coherent optical phenomena:
  - Self-induced transparency (SIT)
    - Two–photon self–induced transparency (TPSIT)
  - Dicke superradiance

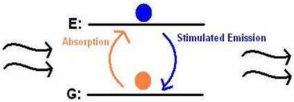
Remainder: SIT and Superradiance



- Group of N ($N \gg 1$) Absorbers or Emitters– “atoms”in ground or excited state–interact with a common light field whose wavelength is much greater than the inter–atomic separation
- SIT–Material which normally absorbs light becomes completely transparent to a short-duration light pulse whose intensity exceeds some critical value.
- Collective emission of the high intensity coherent pulse $I \sim N^2$
- Expected behaviour $I \sim N$

Remainder:SIT

- It is the consequence of the reemission of absorbed radiation in phase with the driving optical field.
- large pulse
- short duration $\tau_p \gg \tau_S$
- single pulse may excite atom and produce stimulated de–excitation



- Area theorem: SIT may occur only if pulse "area" satisfy

$$\int dt E^s(t) = 2\pi\mathcal{N},\ \mathcal{N} = 1,2,3\ldots\ s = 1,2$$

SIT–Features

- Lossless propagation: input energy is equal to the energy of output pulse
    - there is no spontaneous emission: pulse duration is much shorter than the life–time of atom in the excited state
- Reshaping of the pulse: the output pulse may be reshaped by the medium if the input pulse is not SYMMETRIC HYPERBOLIC SECANT
- Pulse is slowed down
    - As light pulse passed through the absorber, it is delayed.
    - for the absorber of length l delay time is: $\tau_D = \frac{l}{v} - \frac{l}{c}$

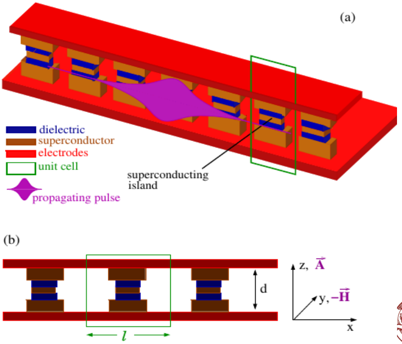
Implications

- Manipulation of light (Yet another?)
    - Slowing down of light.
    - QMMs offer controllable, enhanced slowing down of light
- The experimental confirmation of these effects in QMM may open a new pathway to potentially powerful quantum computing.

Outline

The Model: Experimental setup

- Foundation: QMM model of Rakhmanov, Zagoskin, Savel'ev, Nori, PRB **77** 144507 (2008)
- QMM: Large number of identical SCCQB embedded inside the massive superconductor



Theoretical model

Variables

- $a_n = 2\pi D A_{x,n}/\Phi_0$ - dimensionless EM potential.
- φ_n the superconducting phase on *n*th island.

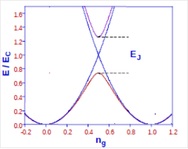
$$H = \sum_n \left[\frac{\dot{\varphi}_n^2}{2} - 2\cos\varphi_n \right] + \sum_n \left[\frac{\dot{\alpha}_n^2}{2} + \beta^2 (\alpha_{n+1} - \alpha_n)^2 \right] + \sum_n \left[2\cos\varphi_n (1 - \cos\alpha_n) \right],$$

Energy parameters

- Energy unit $E_J = \Phi_0 I_c/(2\pi c)$–Josephson energy,
- $\omega_J^2 = 2eI_c/\hbar C$ Josephson frequency
- $\beta^2 = (8\pi D E_J)^{-1}(\Phi_0/2\pi)^2$-dimensionless speed of light,
- $\Phi_0 = h/(2e)$ is the magnetic flux quantum,
- I_c and C is the critical current and capacitance, respectively, of the JJs,
- D is the separation of the superconducting electrodes of the waveguide,
- dimensionless time $\tau = \omega_J t$, with $\omega_J = eI_c/(\hbar C)$ and t being the temporal variable in natural units.

Two-level truncation

- SCCQB Eigenvalues ($E_{p,n}$) and eigenfunctions $\Upsilon_p(\varphi_n)$ are given as solutions of the Mathieu equation $(-\partial^2/\partial\varphi_n^2 + E_{p,n} - 2\cos\varphi)\Upsilon_p(\varphi_n) = 0$



- In the most cases of the interest only two lowest levels are important E_p $(p = 0, 1)$
- Effective two level model

$$H = \sum_{n,p} E_p(n) a^\dagger_{n,p} a_{n,p} + \sum_n \left[\dot{\alpha}_n^2 + \beta^2(\alpha_{n+1} - \alpha_n)^2\right] + 4 \sum_{n,p,p'} V_{p,p'}(n) a^\dagger_{n,p} a_{n,p'} \sin^2 \frac{\alpha_n}{2}$$

$$V_{p,p'}(n) = \int d\varphi_n \Upsilon^*_p(\varphi_n) \cos\varphi_n \Upsilon_p(\varphi_n)$$

Outline

EM radiation in QMM: Evolution equations

- Schrödinger equation (SE) for qubit: We took that qubit is in the superposition state

$$|\Psi_n\rangle = \sum_{n,p=0,1} \Psi_{n,p} a_p^+ |0\rangle$$

$$i\dot{\Psi}_{n,p} = \epsilon_p \Psi_{n,p} + \frac{4E_J}{\hbar\omega_J}\sum_{p'} V_{p,p'}(n)\Psi_{n,p'} \sin^2\frac{\alpha_n}{2}$$

- Maxwell equation for EM wave.

$$\ddot{\alpha}_n = \beta^2(\alpha_{n+1}+\alpha_{n-1}-2\alpha_n) - 2\frac{E_J}{\hbar\omega_J}\sum_{p,p'} V_{p,p'}\Psi^*_{n,p}\Psi_{n,p'}\sin\alpha_n,$$

Approximations and Analytic solutions

- Linearized model: small intensity of EM radiation $\alpha_n \ll 1$
- Slowly varying envelope approximation
- Exact resonance of EM radiation and qubit
- Continuum approximation

Bloch variables

- Maxwell–Bloch equations for amplitude of EM field ($\alpha_n(t)$) and Components of the Bloch vector

$$\ddot{\alpha}_n(t) - \beta_0^2(\alpha_{n+1} + \alpha_{n-1} - 2\alpha_n) - \left[\frac{\Omega^2}{2} + \mu R_3 + D R_3\right]\alpha_n = 0.$$

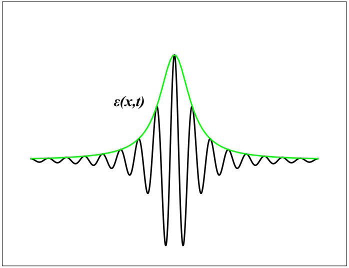
$$\begin{array}{rl} R_1 = |\Psi_1|^2 - |\Psi_0|^2, & \\ R_2 = i(\Psi_1^* \Psi_0 - \Psi_0^* \Psi_1) & \\ R_3 = \Psi_1^* \Psi_0 + \Psi_0^* \Psi_1, & . \end{array} \qquad \begin{array}{rcl} \dot{R}_1 & = & -\mu\alpha_n^2 R_2, \\ \dot{R}_2 & = & -[\Delta + \Omega\alpha_n^2] R_3 + \mu\alpha_n^2 R_1 \\ \dot{R}_3 & = & [\Delta + \Omega\alpha_n^2] R_2. \end{array}$$

where $V_{01} = V_{10}$, $D = (V_{11} - V_{00})E_J/(2\hbar\omega_J)$, $\Omega^2 = (V_{11} + V_{00})E_J/(2\hbar\omega_J)$, $\mu = V_{10}E_J/(\hbar\omega_J)$, and $\Delta = \epsilon_1 - \epsilon_0 \equiv (E_1 - E_0)E_J/(\hbar\omega_J)$.

Slowly varying envelope approximation

- Introduction of slow phase and envelope

$$\alpha(x,t) = \varepsilon(x,t)\cos\Psi(x,t)$$
$$\Psi(x,t) = kx - \omega t + \varphi(x,t)$$



- rotation in abstract space, around R_y axis.

$$\begin{aligned} R_x &= r_x\cos 2\Psi + r_y\sin 2\Psi \\ R_y &= r_y\cos 2\Psi - r_x\sin 2\Psi \end{aligned}$$

Truncated system

$$\dot{\varepsilon} + \frac{\beta^2 k}{\omega}\varepsilon_x = -\frac{\hbar\omega_J}{E_J}\frac{\mu\varepsilon}{2\omega} r_y,$$

$$\dot{\phi} + \frac{\beta^2 k}{\omega}\phi_x = -\frac{\hbar\omega_J}{E_J}\frac{D}{\omega} R_z,$$

$$\dot{r}_x = -(\Delta - 2\omega + 2\dot{\phi} + D\varepsilon^2) r_y,$$

$$\dot{r}_y = (\Delta - 2\omega + 2\dot{\phi} + D\varepsilon^2) r_x - \frac{\mu\varepsilon^2}{2} R_z,$$

$$\dot{R}_z = \frac{\mu\varepsilon^2}{2} r_y.$$

Resonant propagation

- truncated system posses soliton like solutions under the resonance and transparency conditions

$$\Delta = 2\omega,\ k = \pm\frac{\sqrt{\omega^2 - \Omega^2}}{\beta}$$

- lorentzian pulse

$$\varepsilon = \frac{\varepsilon_0}{\sqrt{1 + \frac{\tau^2}{\tau_p^2}}}$$

$$\varepsilon_0^2 = -\frac{4R_0\hbar\omega_J v}{E_J(1+\gamma^2)\omega(c-v)}, \qquad \tau_p = -\frac{E_J\omega(c-v)}{R_0\mu\hbar\omega_J v}\sqrt{1+\gamma^2}, \quad \gamma = \frac{4D}{\mu}$$

Conclusions

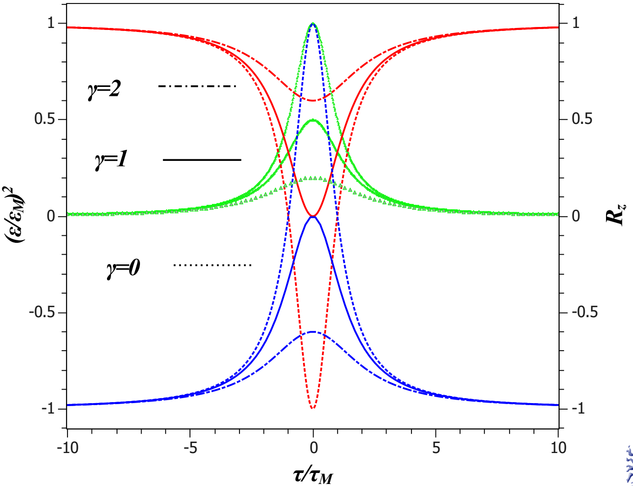
$$v = \beta \sqrt{1 - \frac{\hbar\omega_J}{E_J}\left(\frac{\Omega}{\omega}\right)^2}\left[1 \pm \frac{4\sigma^2\hbar\omega_J}{\varepsilon_0^2\omega E_J}\right]^{-1},$$

$$v = \beta \sqrt{1 - \frac{\hbar\omega_J}{E_J}\left(\frac{\Omega}{\omega}\right)^2}\left[1 \pm \frac{\tau_p\mu\sigma\hbar\omega_J}{\omega E_J}\right]^{-1}, \tag{3.1}$$

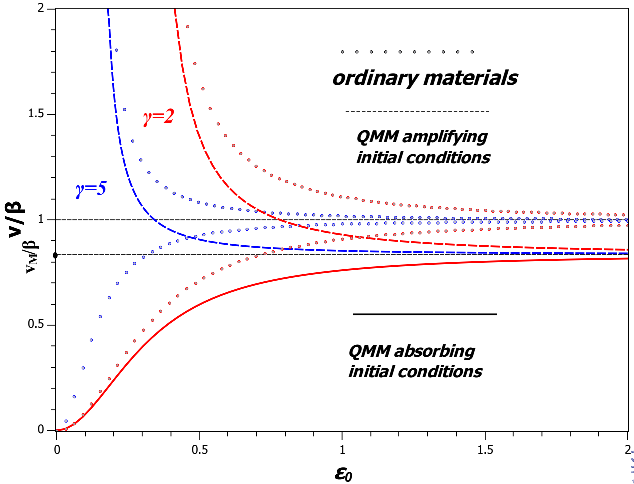
For absorbing QMMs we deduce from Eq. (3.1) that the more intense pulse propagate faster and that there exist an upper bound on the pulse velocity

$$v_0 = \beta \sqrt{1 - 2\frac{V_{11} + V_{00}}{(E_1 - E_0)^2}\left(\frac{\hbar\omega_J}{E_J}\right)^2}, \tag{3.2}$$

results


γ=2
γ=1
γ=0
$(\varepsilon/\varepsilon_M)^2$
R_z
τ/τ_M
1
0.5
0
-0.5
-1
-10
-5
0
5
10

results


2
1.5
1
0.5
0
v_M/β
v/β
0
0.5
1
1.5
2
ε_0
γ=5
γ=2
ordinary materials
QMM amplifying initial conditions
QMM absorbing initial conditions

Influence of the initial conditions

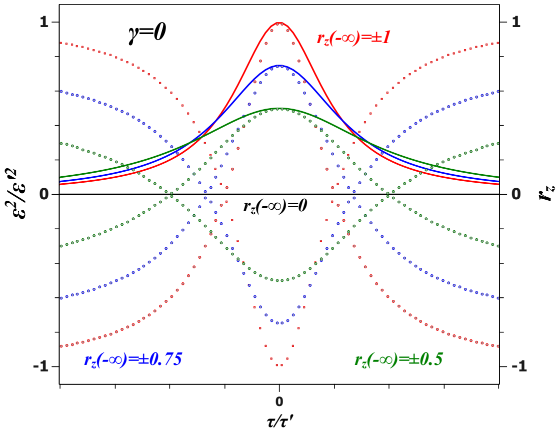


Figure : Illustration of the influence of the initial conditions on the SIT in QMM

Influence of the initial conditions

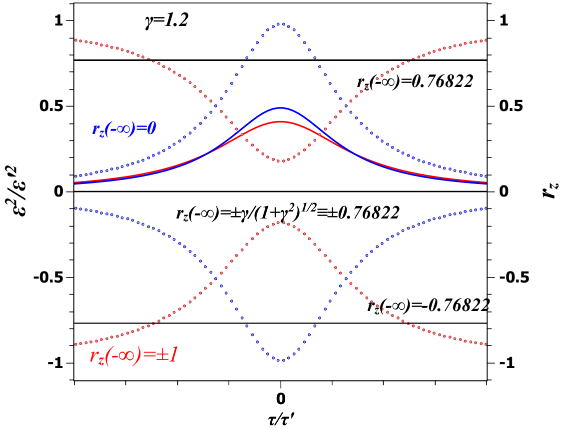
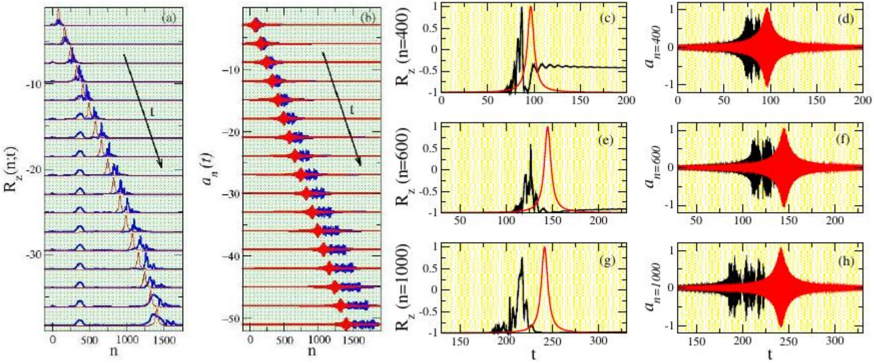


Figure : Illustration of the influence of the initial conditions on the SIT in QMM

Numerical validation



almost coherent pulse propagation for substantial time intervals

Outline

Conclusions

- SIT and superradiance may, in principle, appear in QMM,
- We believe that the predicted effects may aper not only in charge qubits based QMM,
- The benefit–manipulation of light by means of:
  - Tuning of the parameters of the QMM,
  - Choice of the initial conditions

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