General quantum quenches in the transverse field Ising chain

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Novi Sad, 23.12.2015

Non-equilibrium dynamics of isolated quantum systems

1. Consider an isolated many-particle quantum system
2. Take it out of equilibrium
3. Let the system evolve
4. Observe what happens:
    - does the system thermalise/does it relax?
    - how can we describe the steady state?
    - how do the observables behave?

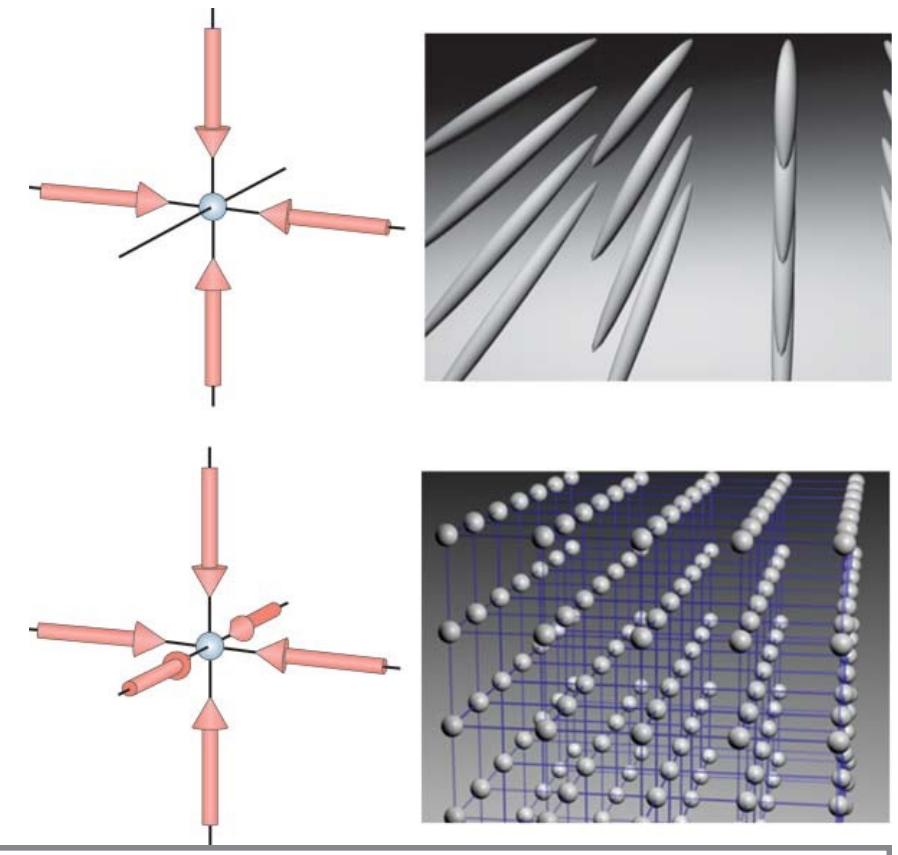
Ultracold atoms in optical lattices

Simulation of solid-state models:

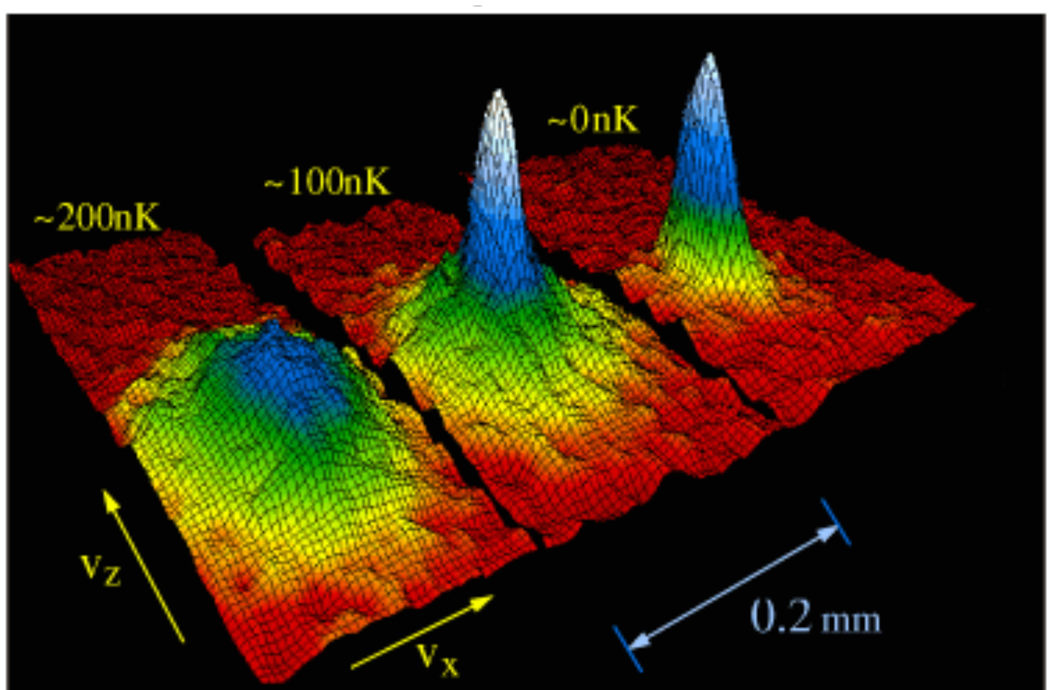
electrons → atoms

crystal lattice → optical lattice

- control of tuneable parameters: lattice type, dimensionality, interactions
- parameters tuneable in time
- almost perfect isolation from the environment

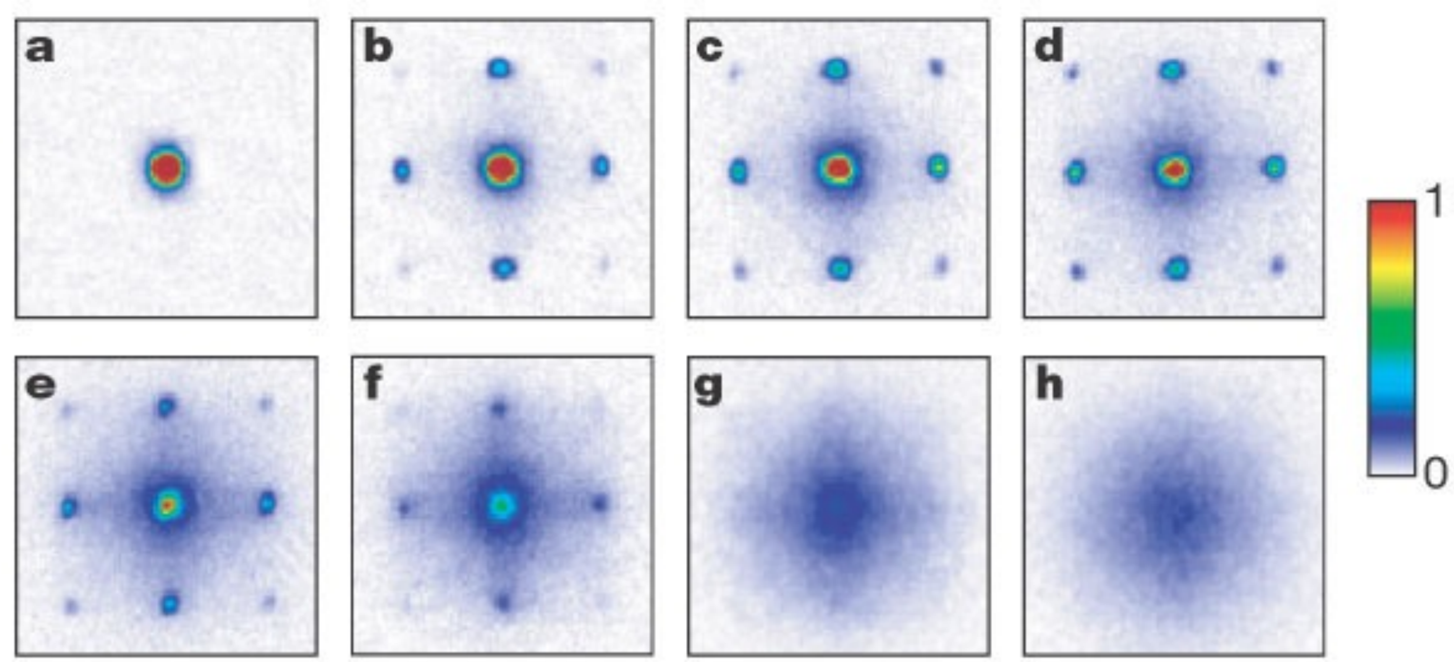


Bloch, Dalibard & Zwerger, RMP, 2008



^{87}Rb BEC

JILA BEC

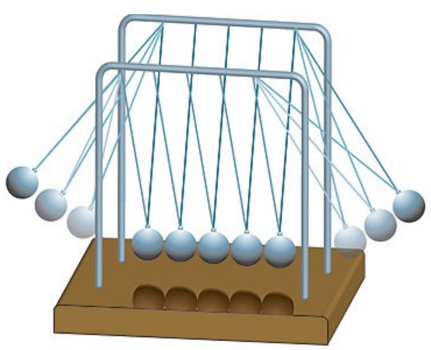


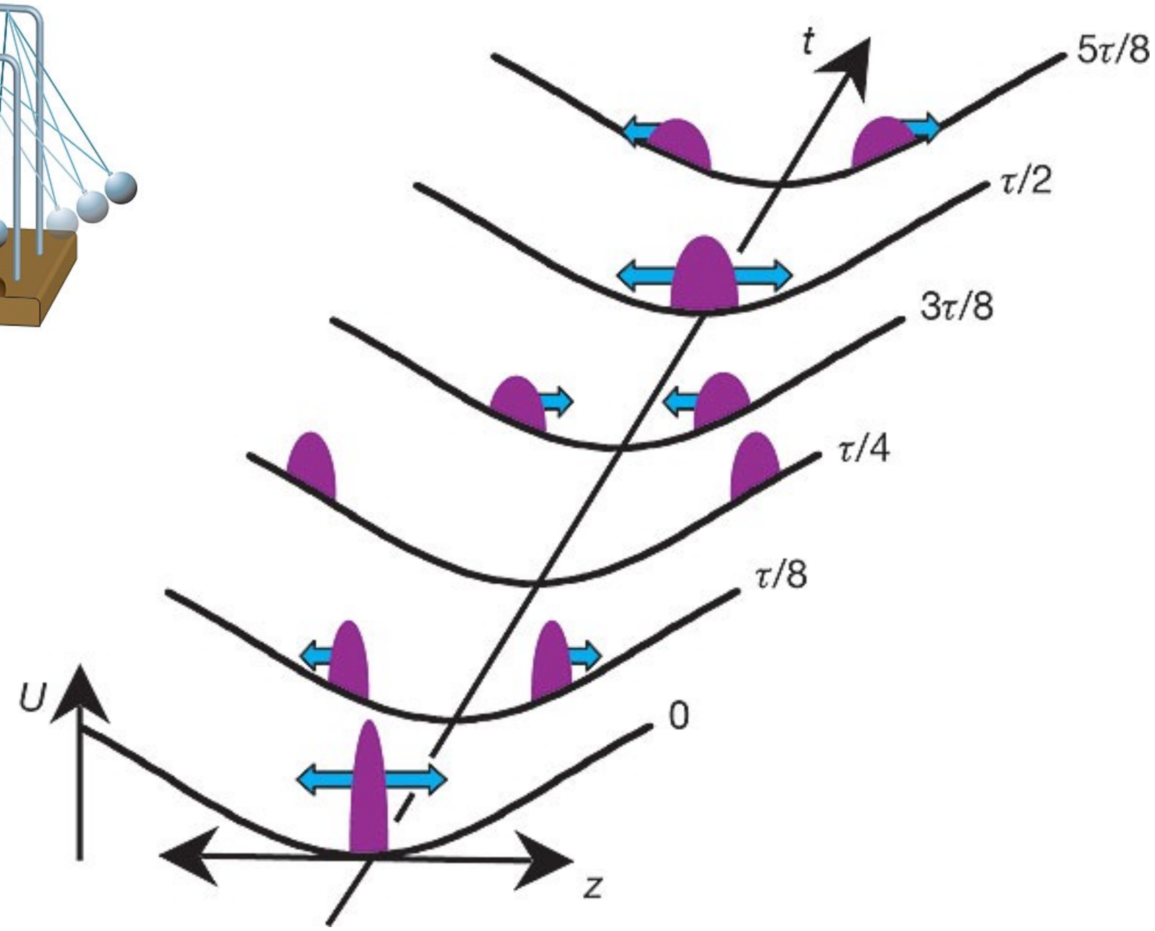
SF to MI transition

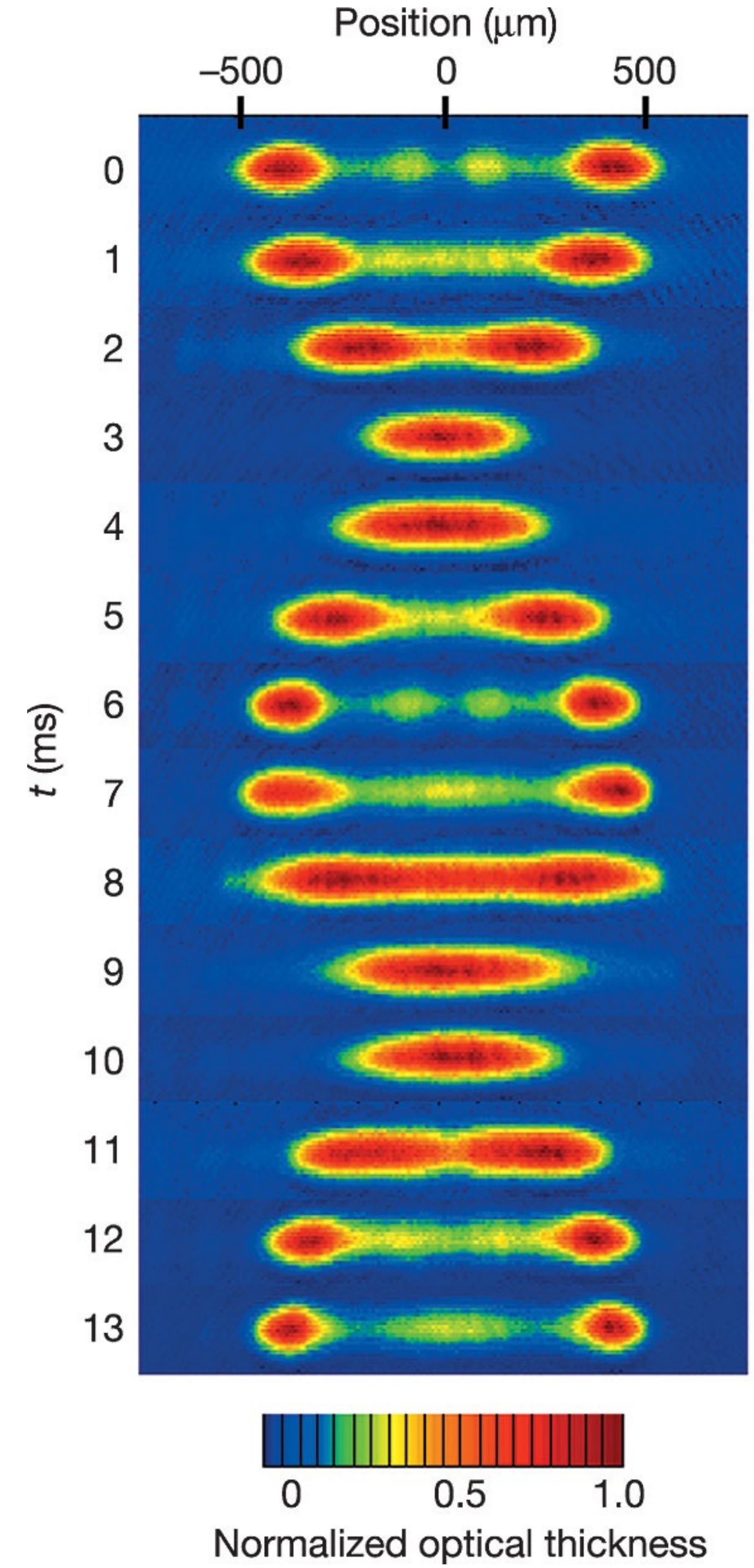
Greiner et al., Nature, 2002

Quantum Newton's cradle

- 1D tubes with 40-250 ^{87}Rb atoms
- initially momentum eigenstate with $\pm k$
- essentially unitary time evolution







Kinoshita, Wenger & Weiss, Nature, 2006

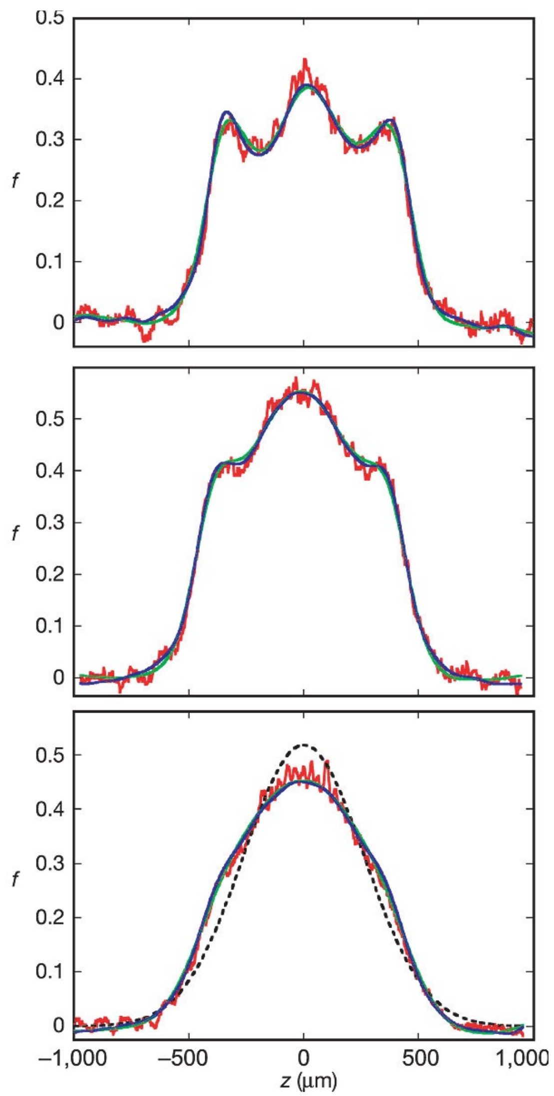
Quantum Newton's cradle

1D

$\gamma = 18$
$t = 15\tau$

$\gamma = 3.2$
$t = 25\tau$

$\gamma = 14$
$t = 25\tau$



- no thermalisation in 1D
- thermalisation after ~3 collisions in 3D

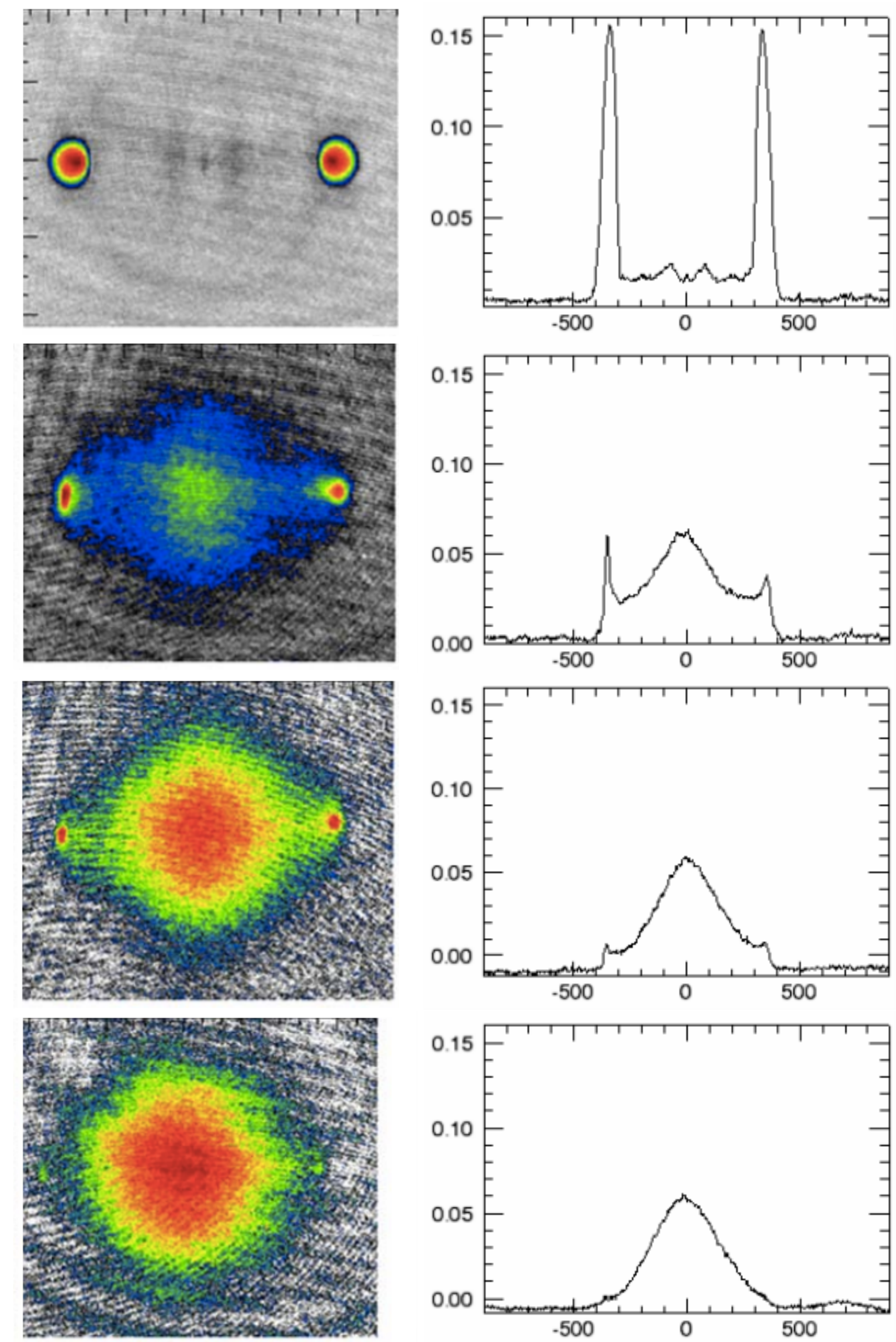
3D

$t = 0\tau$

$t = 2\tau$

$t = 4\tau$

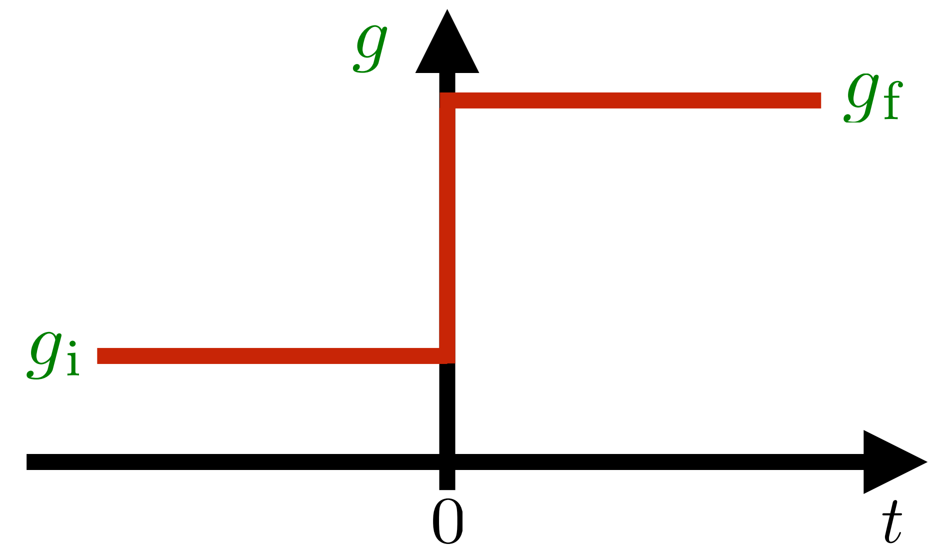
$t = 9\tau$



Kinoshita, Wenger & Weiss, Nature, 2006

Relaxation following sudden quenches

Consider a system $H(g)$ with a tuning parameter g



- pre quench: $H(g_\mathrm{i}),\ |\Psi(0)\rangle$
- quench: $g_\mathrm{i} \to g_\mathrm{f}$
- post quench: $H(g_\mathrm{f}),\ |\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H(g_\mathrm{f})t}|\Psi(0)\rangle$

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H(g_\mathrm{f})t}|\Psi(0)\rangle = \sum_n \langle n|\Psi(0)\rangle \mathrm{e}^{-\mathrm{i}E_n t}|n\rangle$$

decomposition in terms of post-quench Hamiltonian eigenstates

Observables:

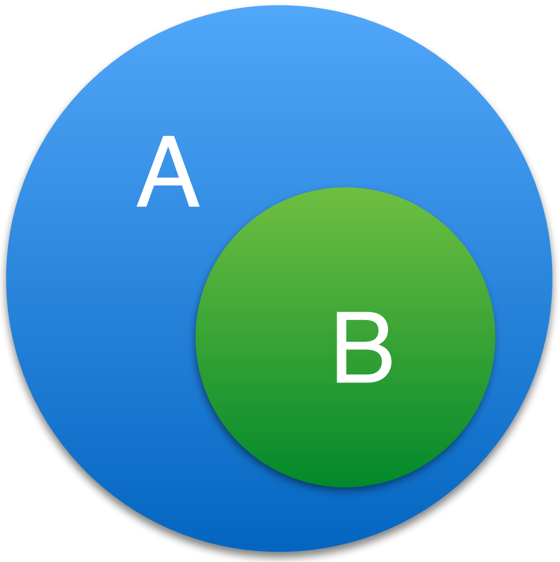
$$\langle\Psi(t)|O|\Psi(t)\rangle = \sum_{m,n}\langle\Psi(0)|m\rangle\langle n|\Psi(0)\rangle\langle n|O|m\rangle \mathrm{e}^{\mathrm{i}(E_n - E_m)t}$$

interference of oscillating phases

Relaxation following quenches

Full system **A**∪**B**:

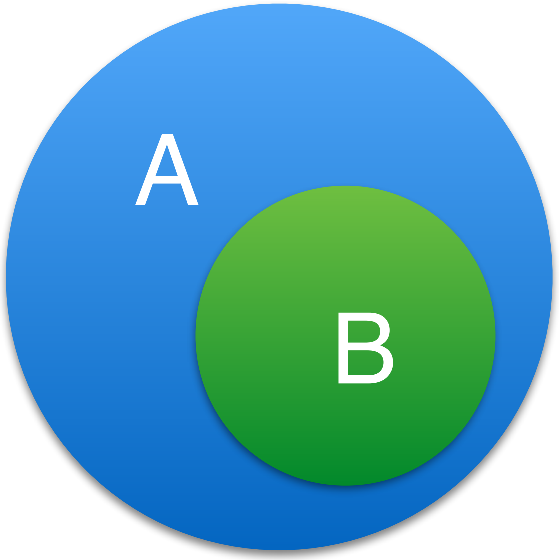
- initial state is a pure state $|\Psi\rangle$
  ➡ the system never relaxes as a whole
- density matrix $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$
- observables $\langle\Psi(t)|O|\Psi(t)\rangle = \mathrm{tr}\left[\rho(t)O\right]$



Subsystem **B**:

- **A** (infinite) acts as a bath for **B** (finite)
  ➡ the system relaxes locally
- density matrix $\rho_B(t) = \mathrm{tr}_A \rho(t)$
- observables $$\lim_{t\to\infty}\lim_{L\to\infty}\frac{\langle\Psi(t)|O_B|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)\rangle} = \mathrm{const}$$

Relaxation following quenches



Can we express $\rho_B(\infty)$ in terms of a statistical ensemble?

Generic systems → thermalisation

- energy is the only local integral of motion
- Gibbs ensemble $\rho_{\mathrm{G}} = \frac{1}{Z_{\mathrm{G}}}\exp\big(-\beta H\big)$

Deutsch, PRA, 1991

Integrable systems → relaxation

- additional local integrals of motion
- generalised Gibbs ensemble

Rigol et al., PRL, 2007

$$\rho_{\mathrm{GGE}} = \frac{1}{Z_{\mathrm{GGE}}}\exp\left(-\sum_n \lambda_n I_n\right)$$

Lagrange multipliers

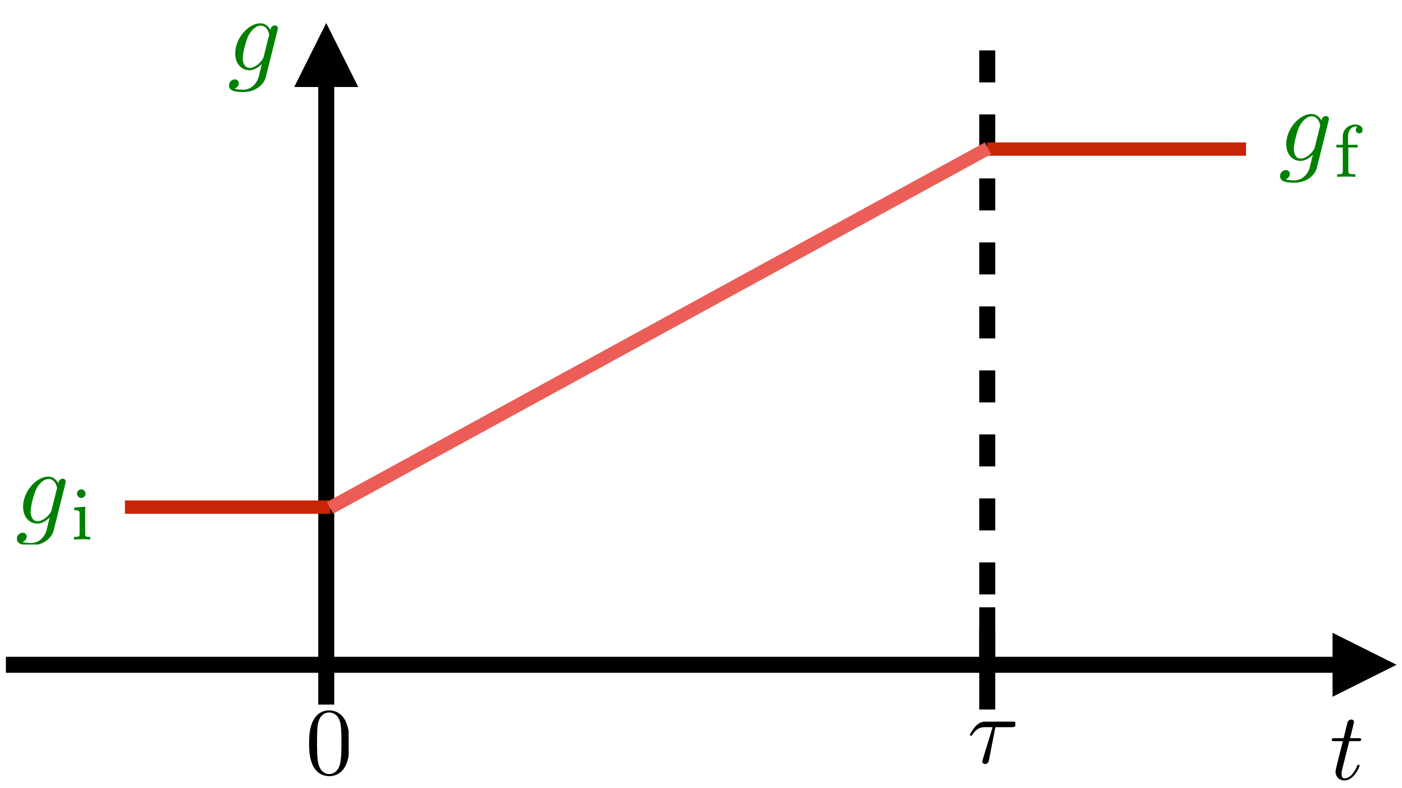
full set of local integrals of motion

$$\mathrm{tr}\left(\rho_{\mathrm{GGE}} I_n\right) = \langle\Psi_0|I_n|\Psi_0\rangle$$

$$[H, I_n] = [I_m, I_n] = 0$$

General quantum quenches

Consider a system $H(g)$ with a tuning parameter g



Initial system: $t < 0$

$$H(g_\mathrm{i})$$

$$|\Psi_0\rangle$$

Post-quench system: $t > \tau$

$$H(g_\mathrm{f})$$

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H(g_\mathrm{f})(t-\tau)}|\Psi(\tau)\rangle$$

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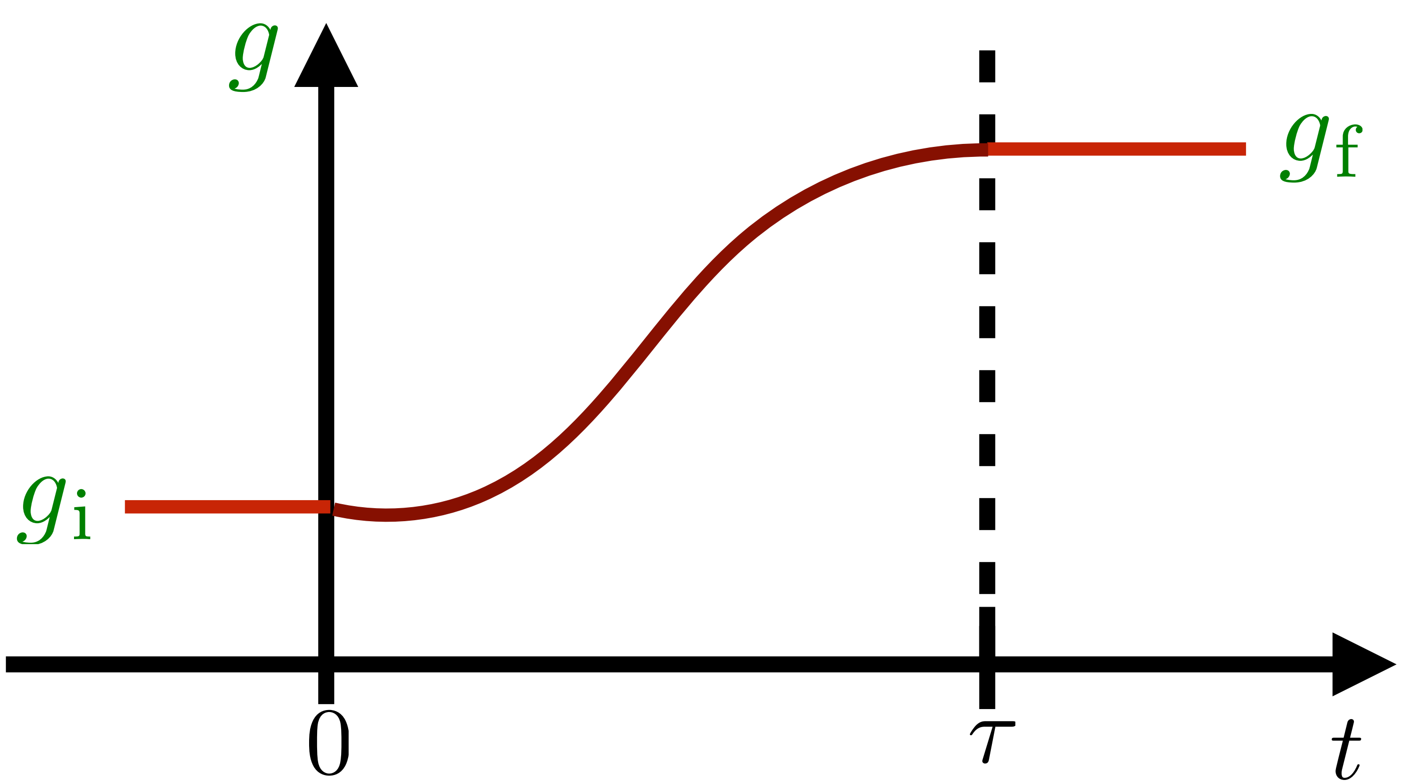

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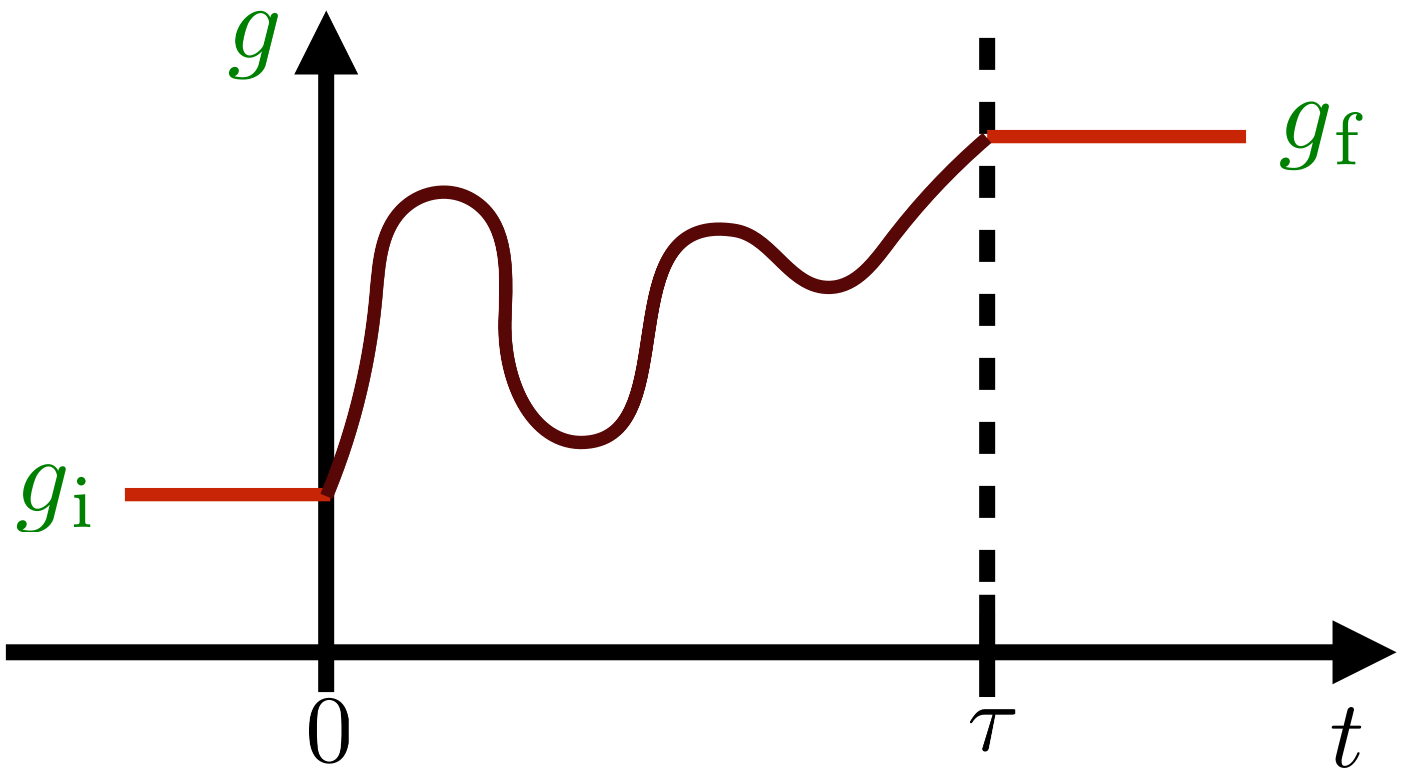

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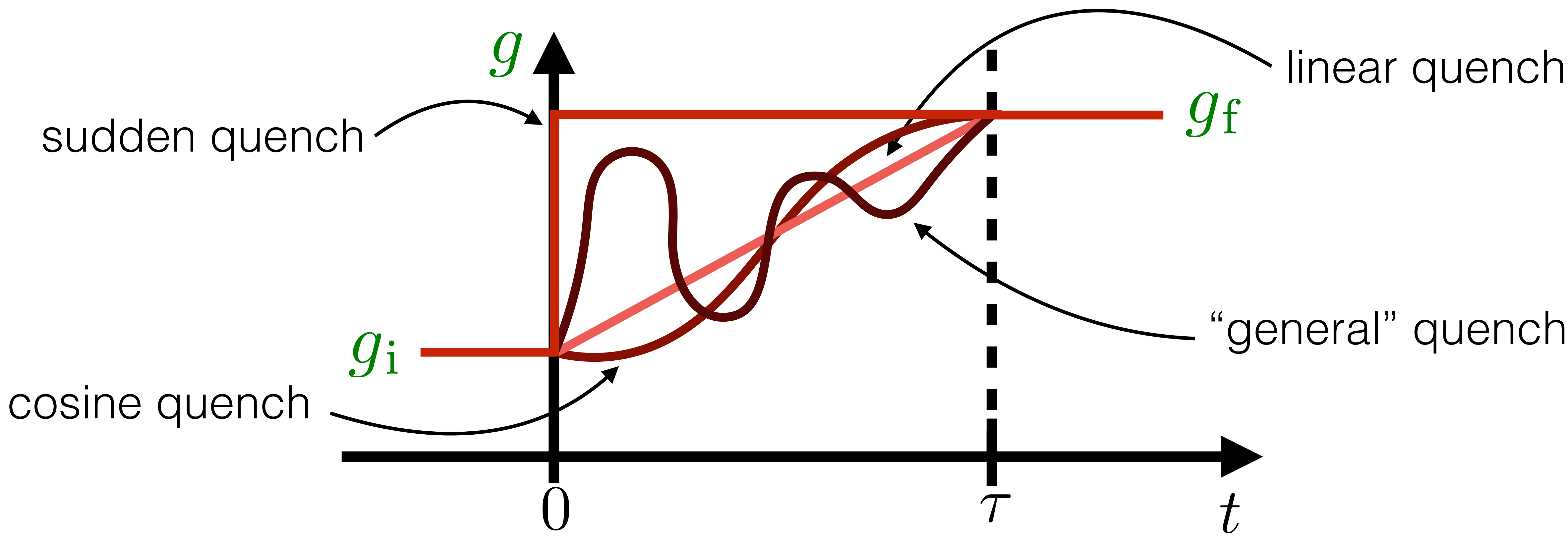

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$H(g_{\mathrm{f}})$

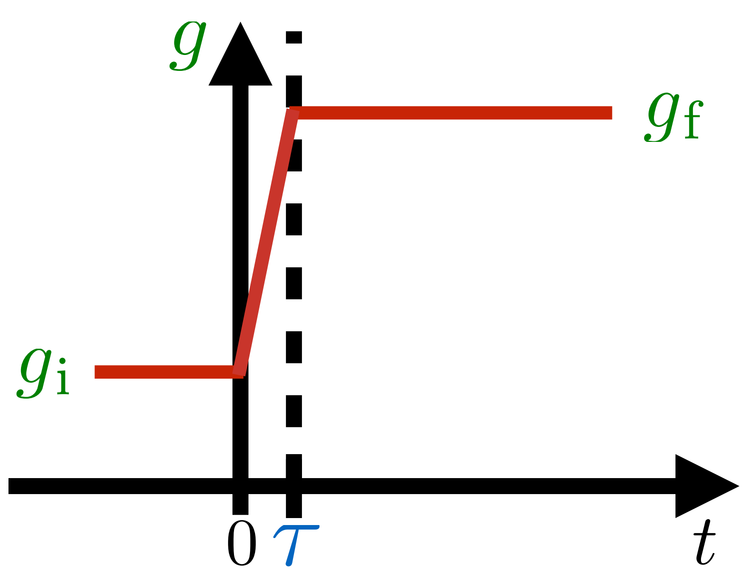
$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H(g_{\mathrm{f}})(t-\tau)}|\Psi(\tau)\rangle$

General quantum quenches

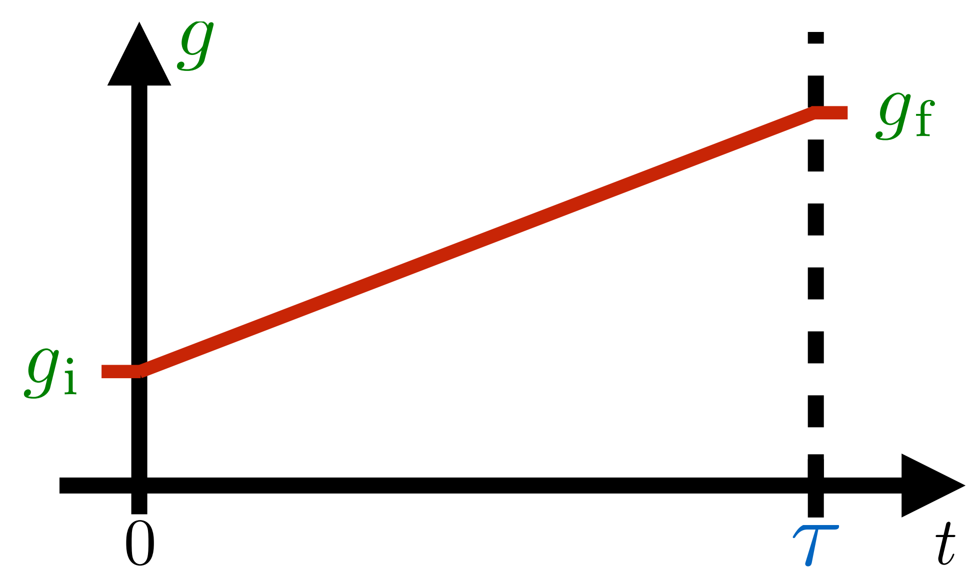
Compare the duration of quench τ to the timescales in the system:

gap in the system, interaction energies

$\tau \to 0$



$\tau \to \infty$

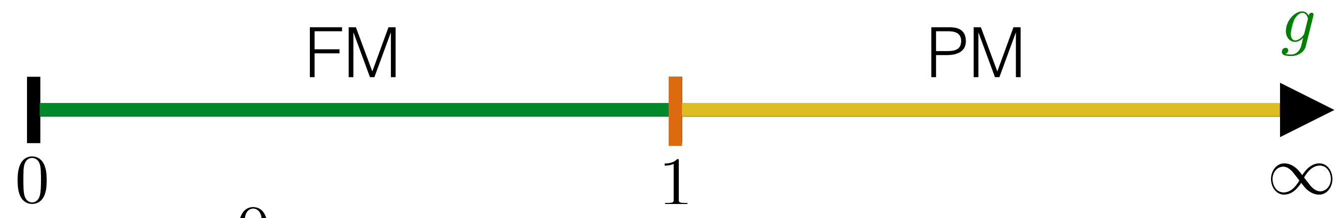


- sudden limit
- the change is too fast for the system to react in intermediate times
- it only notices the final value of the quench parameter

- adiabatic limit
- the change is slow enough for the system to follow in the ground state of each intermediate Hamiltonian

Transverse Field Ising chain

$$H = -J\sum_{i=1}^{N}\left(\sigma_i^x\sigma_{i+1}^x + g\sigma_i^z\right)$$



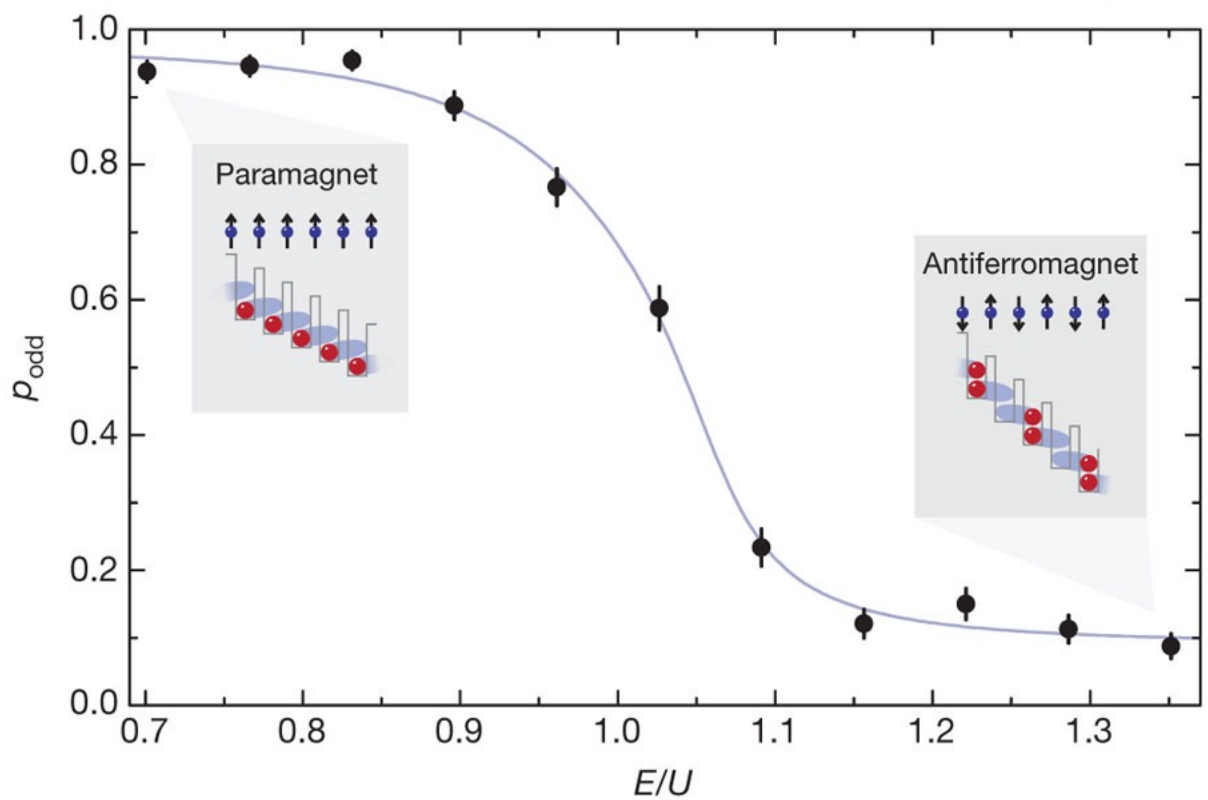
$g = 0:$

$|\downarrow\downarrow\downarrow\dots\downarrow\rangle$

$|\uparrow\uparrow\uparrow\dots\uparrow\rangle$

$g = \infty:$

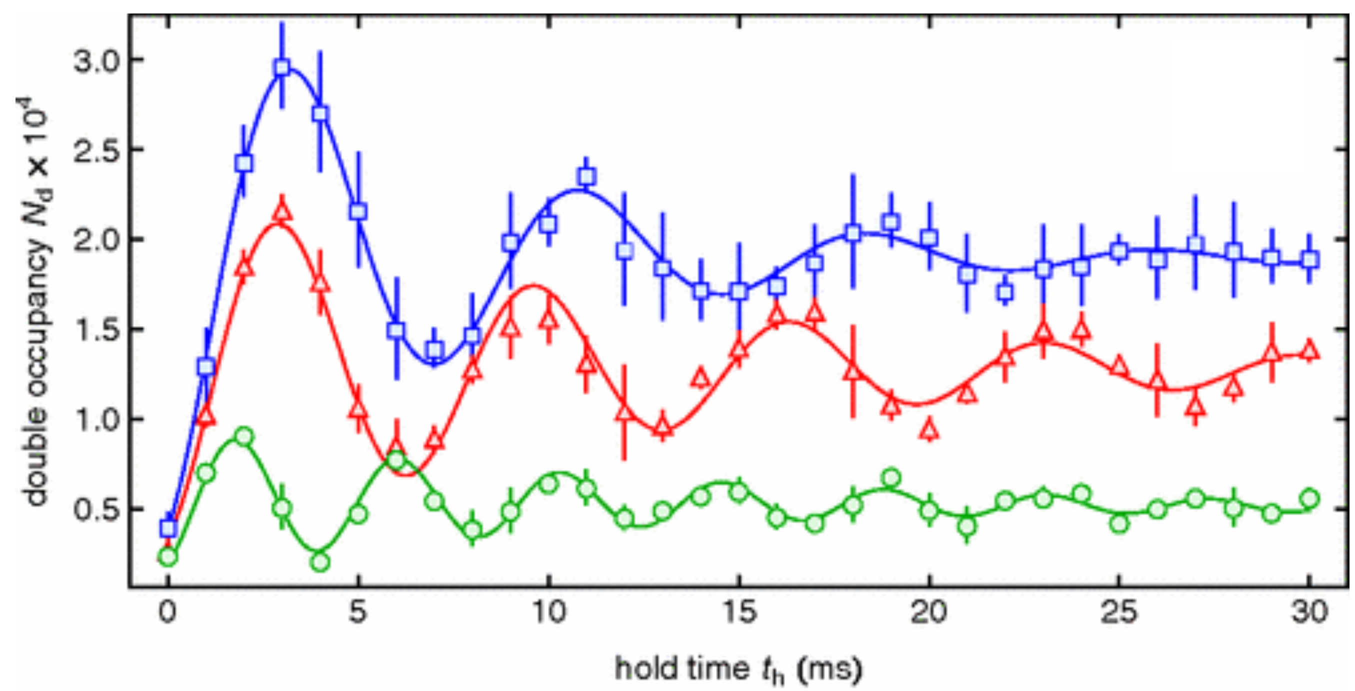
$|\rightarrow\rightarrow\rightarrow\cdots\rightarrow\rangle$



PM to AFM transition in an ultracold atom system

Simon et al., Nature, 2011

Sachdev, Sengupta & Girvin, PRB, 2002



Response of the system to a quantum quench

Meinert et al., PRL, 2013

Transverse Field Ising chain

$$H = -J \sum_{i=1}^{N} \left(\sigma_i^x \sigma_{i+1}^x + g \sigma_i^z \right)$$

1) Jordan-Wigner transformation

$$\sigma_i^z = 1 - 2c_i^\dagger c_i$$

$$\sigma_i^x = \prod_{j<i} (1 - 2c_j^\dagger c_j)(c_i^\dagger + c_i)$$

2) Fourier transformation

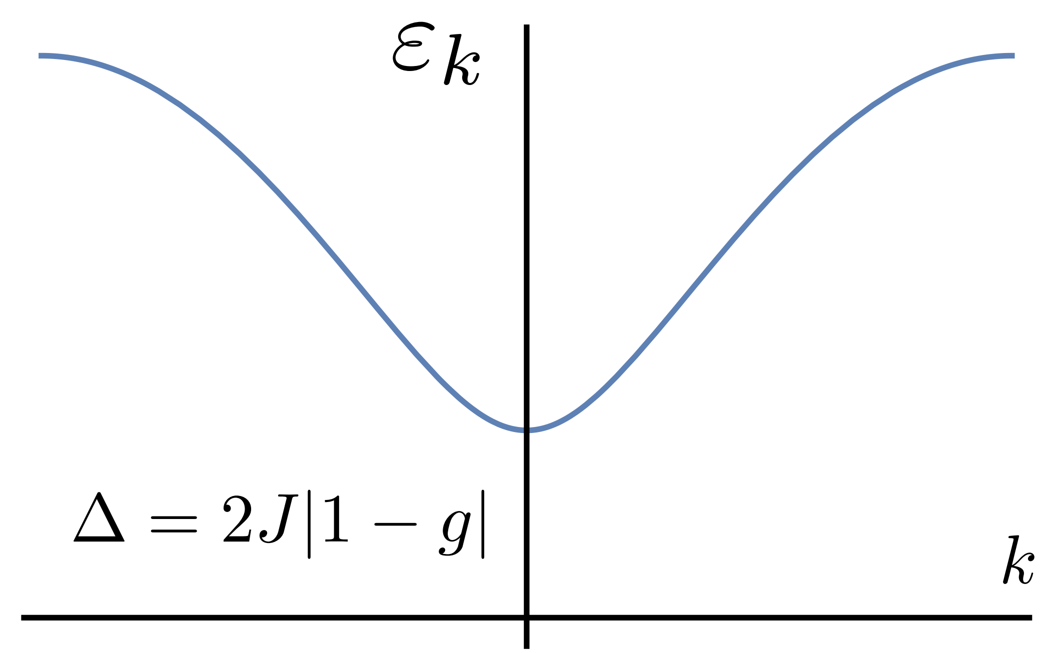
3) Bogoliubov transformation

$$\eta_k = u_k c_k - \mathrm{i} v_k c_{-k}^\dagger$$

$$\eta_k^\dagger = u_k c_k^\dagger + \mathrm{i} v_k c_{-k}$$

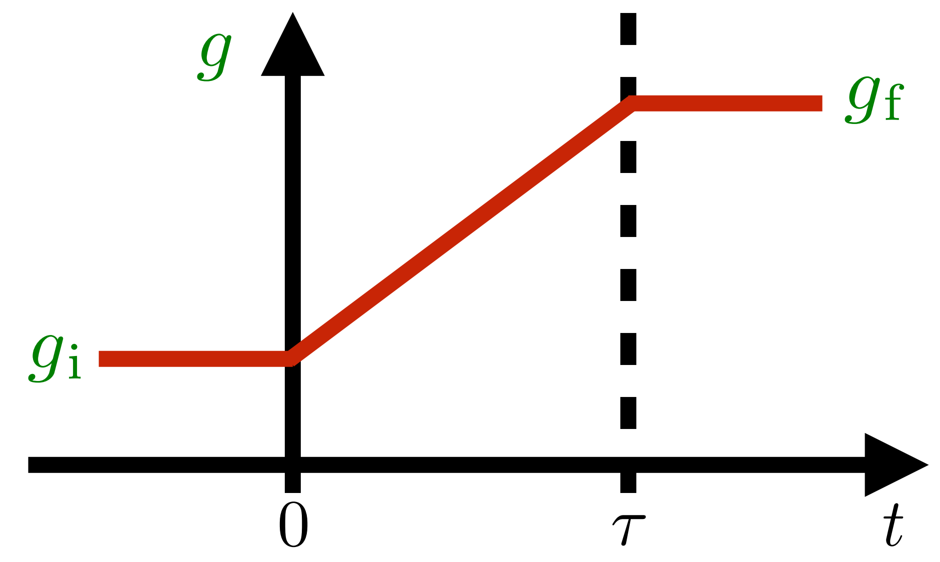
$$\boxed{H = \sum_k \varepsilon_k \left(\eta_k^\dagger \eta_k - \frac{1}{2} \right)}$$

$$\varepsilon_k = 2J\sqrt{1 + g^2 - 2g\cos k}$$



Global general quench in TFI chain

$$\boxed{H(t) = -J\sum_{i=1}^{N}\left(\sigma_i^x\sigma_{i+1}^x + g(t)\sigma_i^z\right)}$$



Each instantaneous Hamiltonian can be diagonalised as

$$H(t) = \sum_k \varepsilon_{k,t}\left(\eta_{k,t}^\dagger\eta_{k,t} - \frac{1}{2}\right)$$

$$\eta_{k,t} = u_{k,t}c_{k,t} - \mathrm{i}v_{k,t}c_{-k,t}^\dagger$$

$$\varepsilon_{k,t} = 2J\sqrt{1+(g(t))^2 - 2g(t)\cos k}$$

How to relate the infinitely many instantaneous Hamiltonians?

Global general quench in TFI chain

Cast the dynamics into the Bogoliubov coefficients: $c_k(t) = u_k(t)\eta_k + \mathrm{i}v_k(t)\eta^\dagger_{-k}$

Explicit time dependance from Heisenberg equations of motion:

$$\mathrm{i}\frac{\partial}{\partial t}c_k(t) = [c_k,\, H]$$

$$\mathrm{i}\frac{\partial}{\partial t}c^\dagger_k(t) = [c^\dagger_k,\, H]$$

$$\boxed{\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} u_k(t) \\ v^*_{-k}(t)\end{pmatrix} = \begin{pmatrix} A_k(t) & B_k \\ B_k & -A_k(t)\end{pmatrix}\begin{pmatrix} u_k(t) \\ v^*_{-k}(t)\end{pmatrix}}$$

$$A_k(t) = 2\big(g(t) - \cos k\big)$$

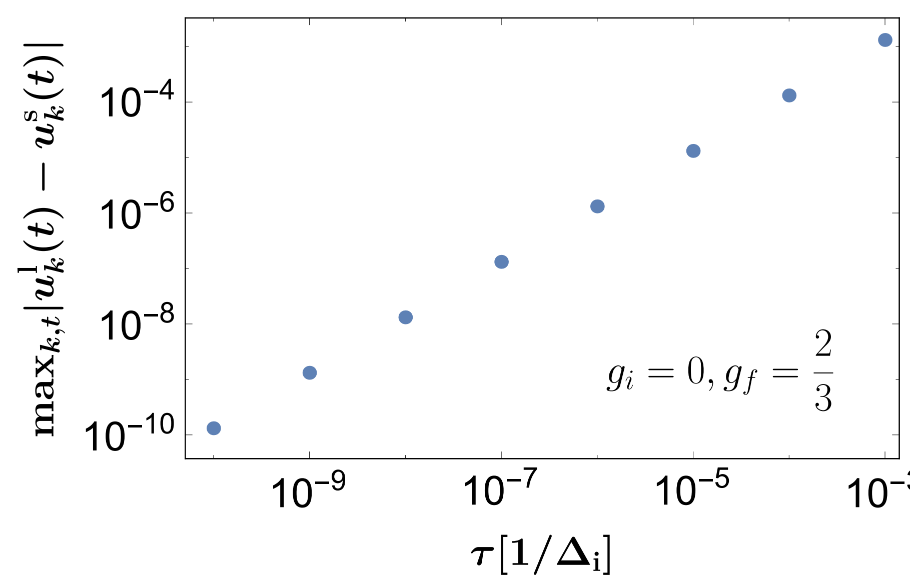
$$B_k = 2\sin k$$

During quench

$$\frac{\partial^2}{\partial t^2}y(t) + \left[A_k(t)^2 + B_k^2 \pm \mathrm{i}\frac{\partial}{\partial t}A_k(t)\right]y(t) = 0$$

Post quench

$$\frac{\partial^2}{\partial t^2}y(t) + \omega_k^2 y(t) = 0$$

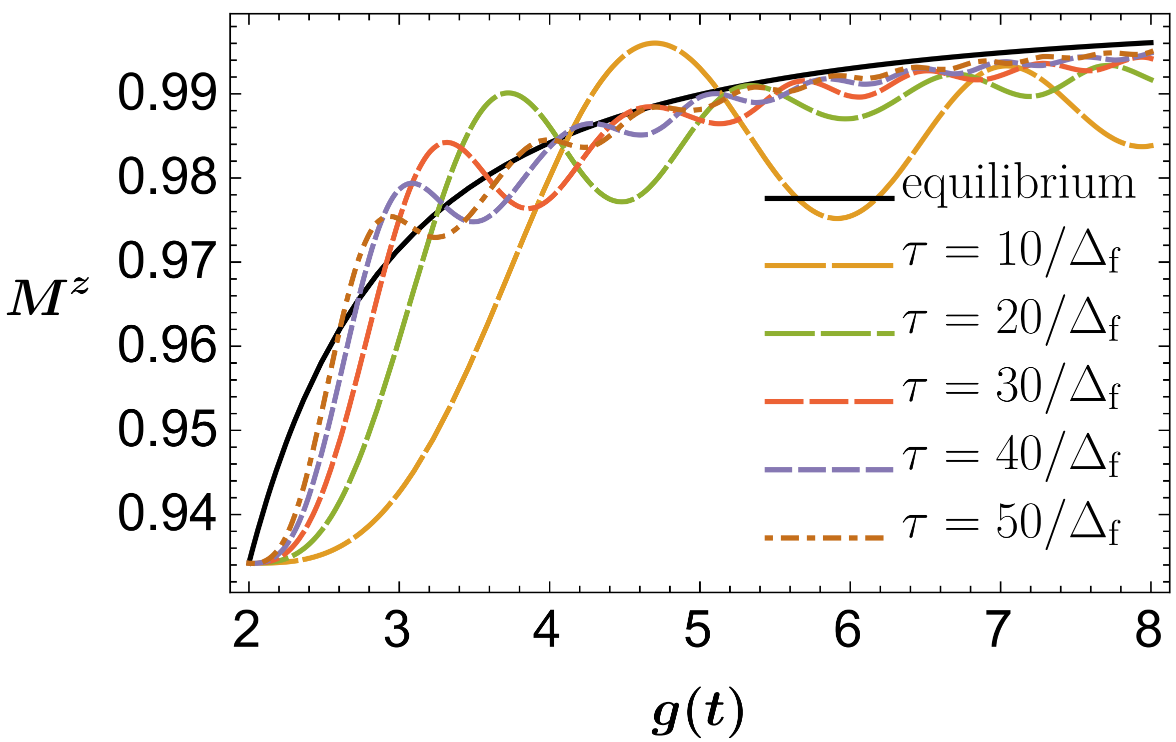


sudden vs linear quench

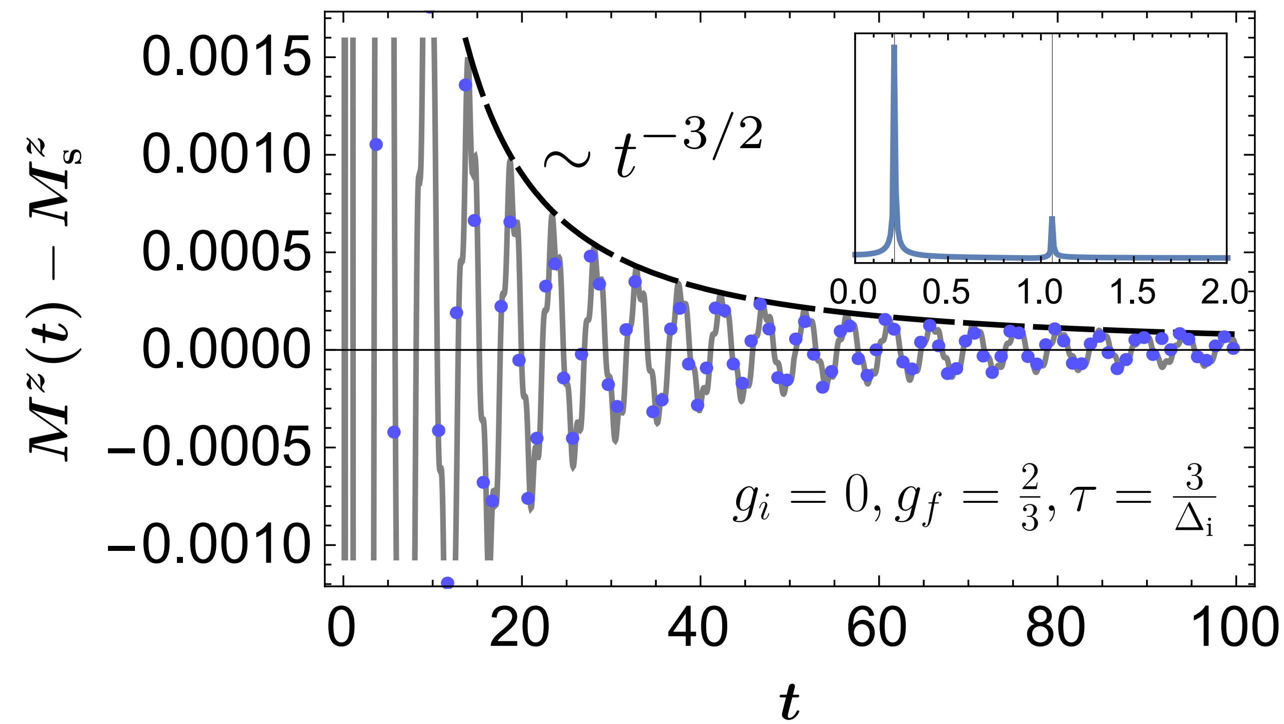
Transverse magnetisation

$$M^z(t) = \langle \sigma^z(t) \rangle$$

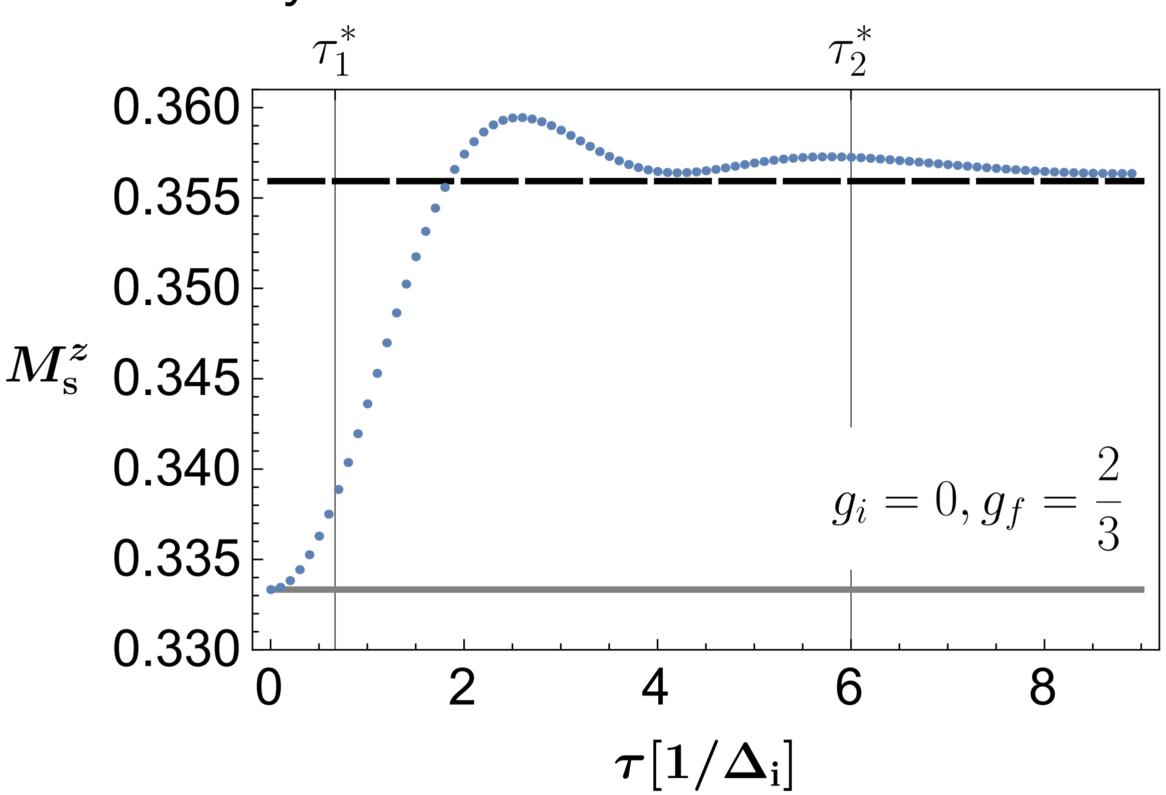
During quench



Post quench



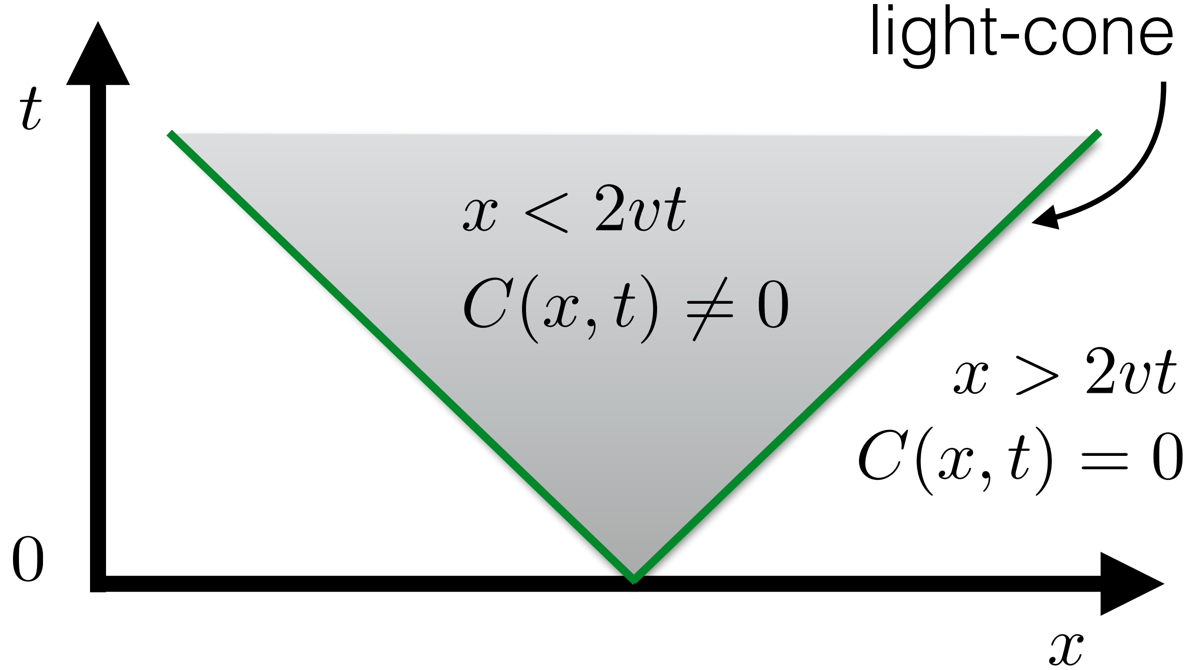
Stationary values

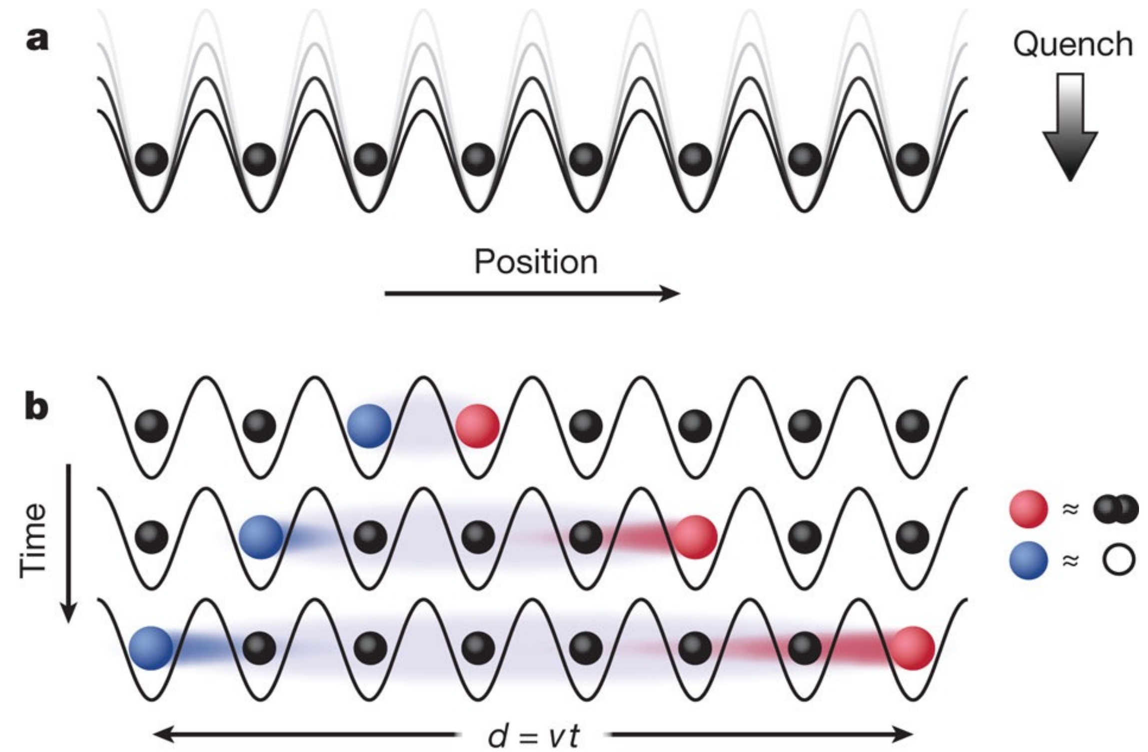


- behaviour during the quench depends on the quench duration
- behaviour after the quench general - algebraic decay with internal oscillations
- GGE constructed from the post-quench mode occupation numbers

Two-point correlation functions

$$C(x,t) = \langle O(x,t)O(0,t)\rangle - \langle O(t)\rangle^2$$





Cheneu et al, Nature, 2012

- initially correlations only within the system correlation length ξ_0
- state $|\Psi_0\rangle$ acts as a source of entangled quasiparticles
- quasiparticles propagate classically through the system
- onset of correlations at $t^* \approx \dfrac{x}{2v}$

Calabrese & Cardy, PRL, 2006

Two-point correlation functions

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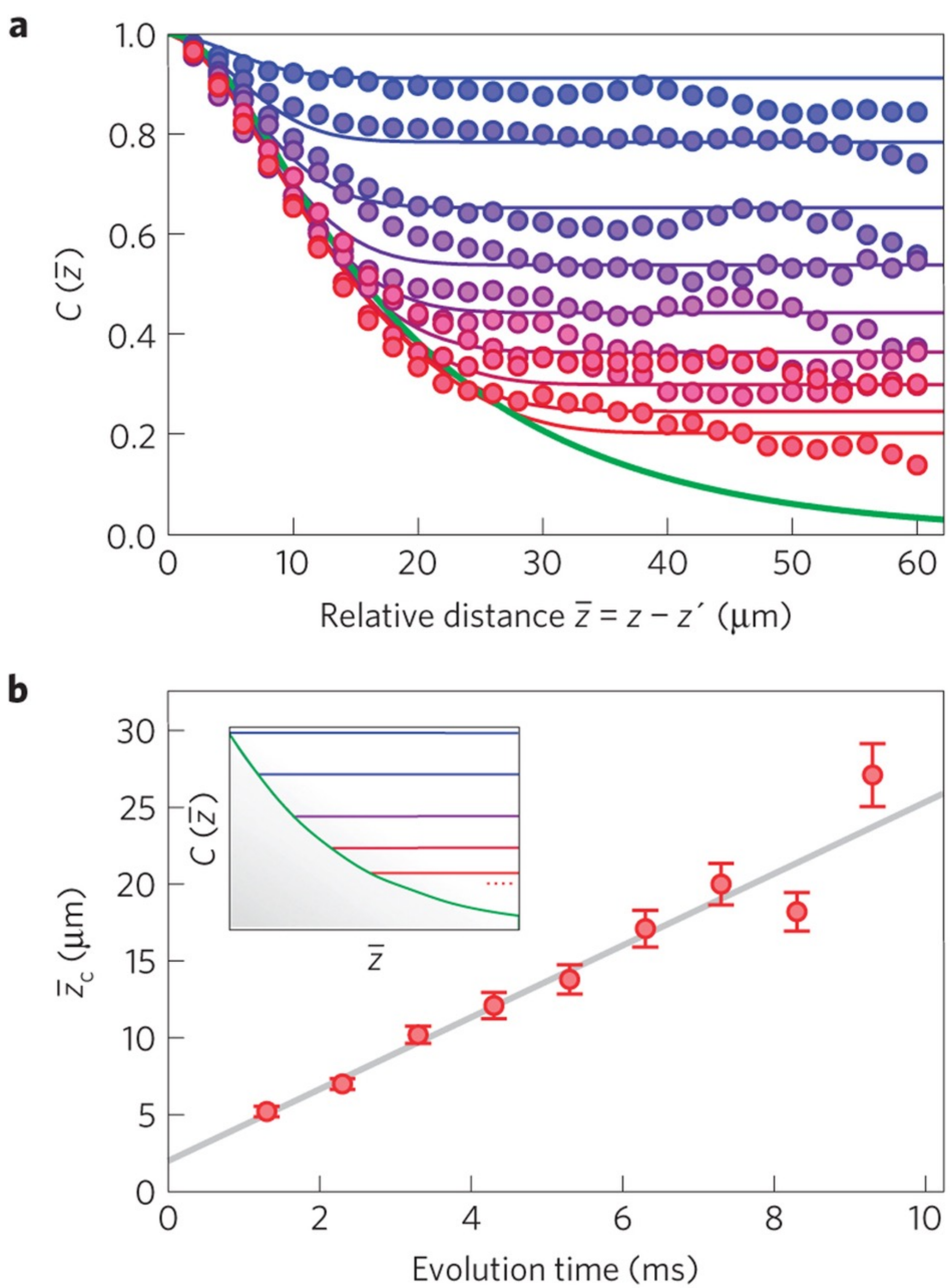
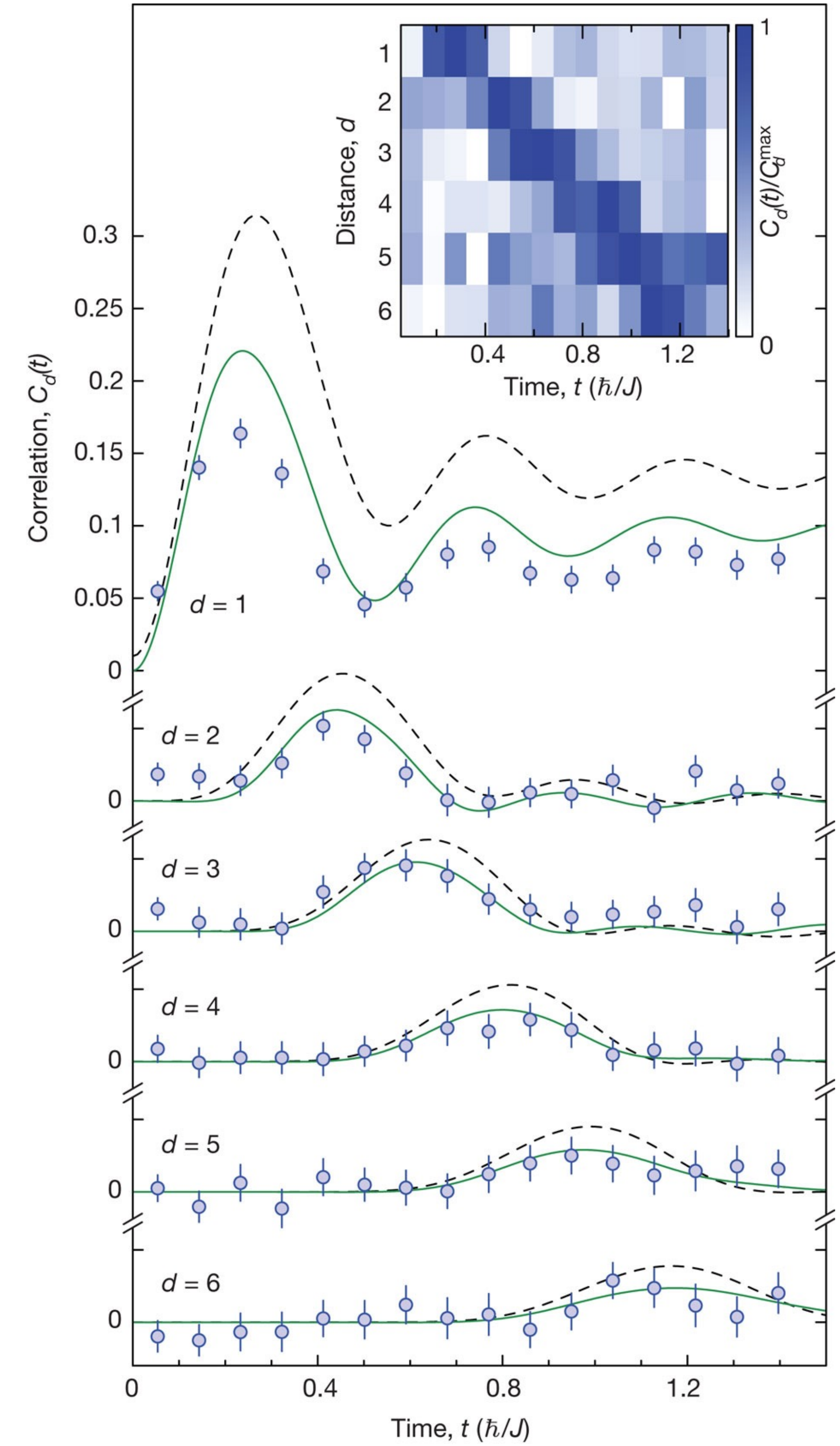


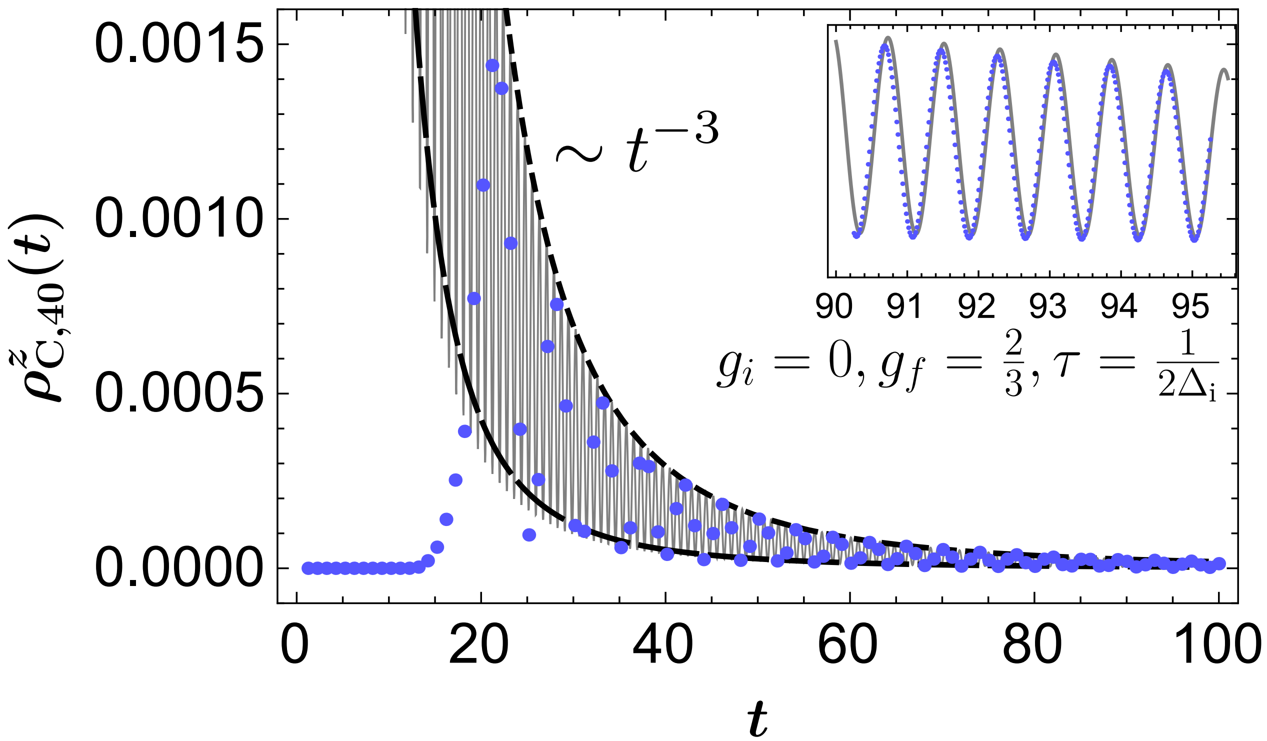


Langen et al, Nature Phys 2013

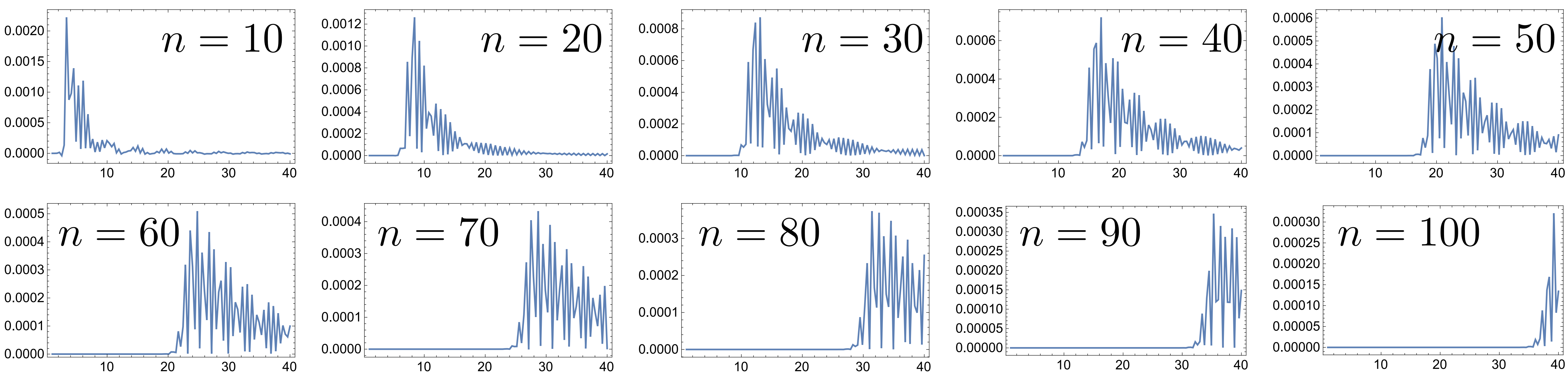
Cheneu et al, Nature, 2012

Transverse two-point correlation function

$$\rho^z_n(t) = \langle \sigma^z_i(t)\sigma^z_{i+n}(t)\rangle$$



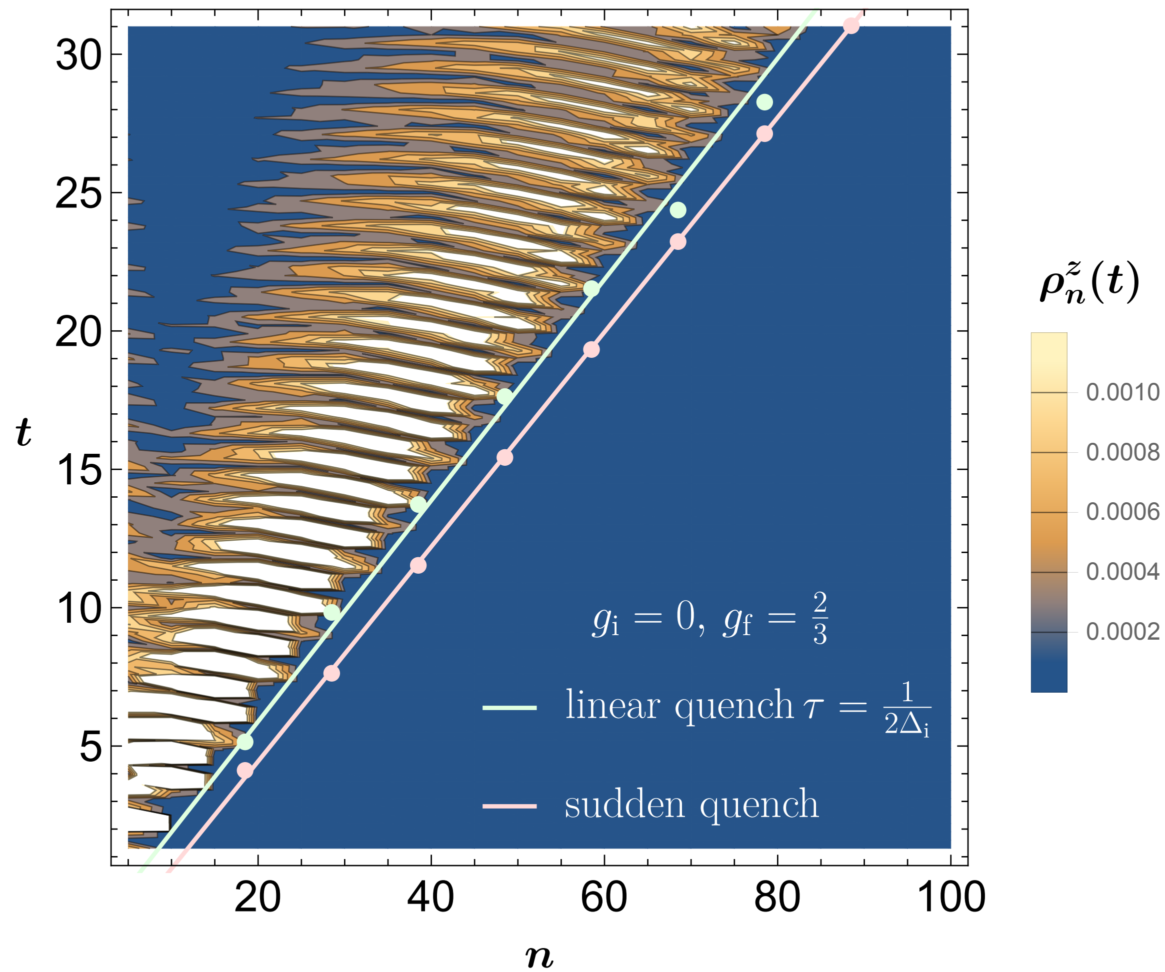
- algebraic decay with internal oscillations
- light-cone effect and Lieb-Robinson bound observable

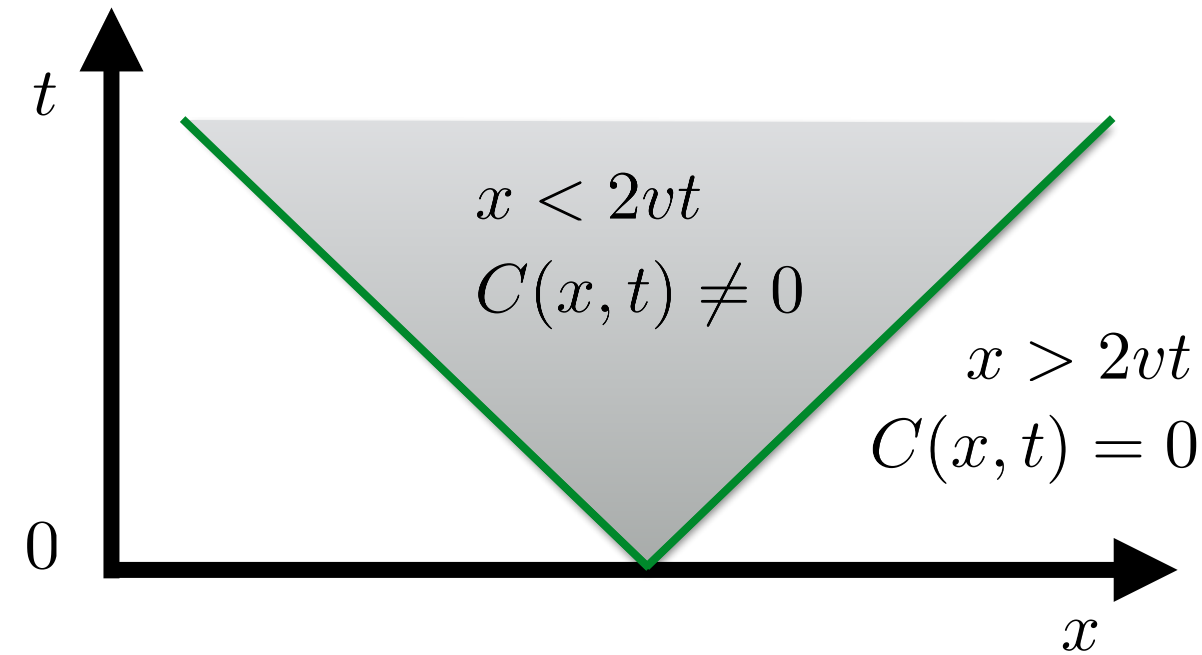


Time evolution of the transverse two-point correlation function for various separations

Transverse two-point correlation function

$$\rho_n^z(t) = \langle \sigma_i^z(t)\sigma_{i+n}^z(t) \rangle$$





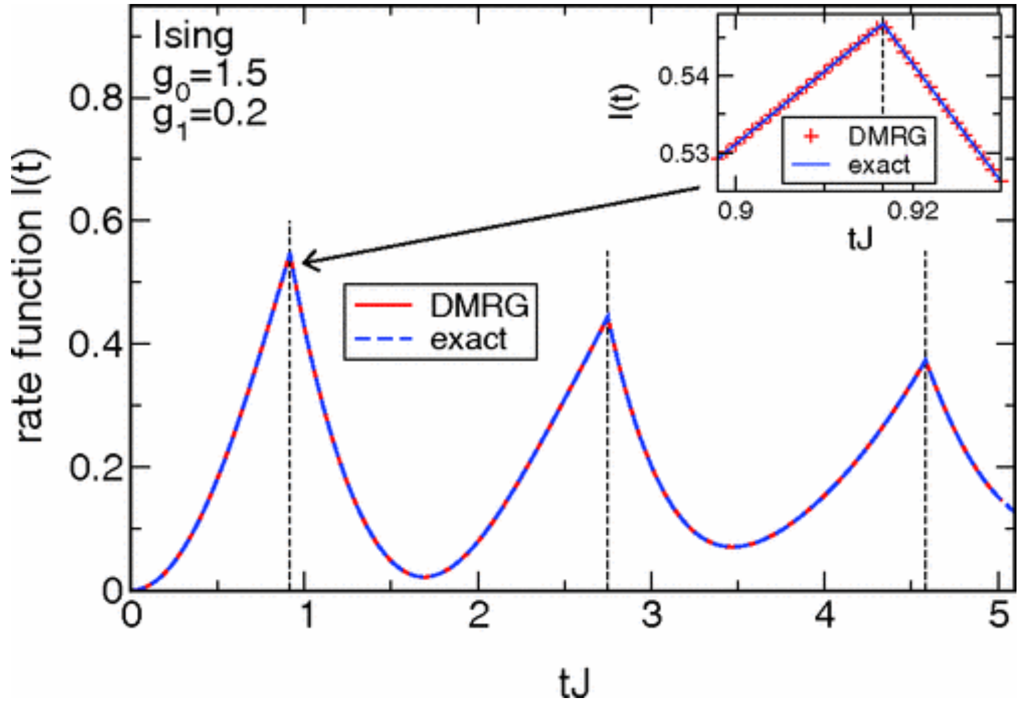
$$C(x,t) = \langle O(x,t)O(0,t)\rangle - \langle O(t)\rangle^2$$

Modification of the quasiparticle picture:

- at any time t during the quench the quasiparticles propagate with their instantaneous velocity
- for general quenches the quasiparticles will not be created just at $t = 0$ but over the quench time τ

Summary and outlook

- Isolated 1D systems show non-trivial non-equilibrium dynamics
- Local observables in the system relax (GGE)
- Two-point functions allow for observation of the light-cone effect
- General quenches add additional features compared to the sudden case
- TFIC: quenching across the critical point (Kibble-Zurek mechanism, DPT)



Karrasch & Schuricht, PRB, 2013