

# *Ab initio* proučavanje neadijabatskih efekata kod malih molekula

Marko Mitić

Fakultet za fizičku hemiju, Univerzitet u Beogradu

Seminar iz fizike/astrofizike  
Departman za fiziku, PMF, Novi Sad  
15. april 2016.



"Fizički zakoni na kojima počiva veliki deo fizike i **cela** hemija su, dakle, potpuno poznati i problem je **samo** u tome što egzaktna primena tih zakona vodi do jednačina koje su isuviše komplikovane da bi se mogle rešiti."

— P. A. M. Dirac, 1929.

# Sadržaj izlaganja

- 1 Kvantno-hemijski problem
  - Šredingerova jednačina
  - Born-Openhajmerova aproksimacija
  - *Ab initio* metode
- 2 Mali molekuli
- 3 Rener-Telerov efekat - opšta razmatranja
- 4 Model za tretiranje R-T efekta kod četvoroatomskih molekula
  - Modelni hamiltonijan
  - Elektronske bazne funkcije
  - Elektronske bazne funkcije - dijabatizacija
  - Mala odstupanja od linearnosti
  - Ugao  $\tau$
  - S-O matični elementi
  - Vibronska baza
  - Rezultati
- 5 Model za molekule sa proizvoljnim brojem jezgara

# Sadržaj izlaganja

- 1 Kvantno-hemijski problem
  - Šredingerova jednačina
  - Born-Openhajmerova aproksimacija
  - *Ab initio* metode
- 2 Mali molekuli
- 3 Rener-Telerov efekat - opšta razmatranja
- 4 Model za tretiranje R-T efekta kod četvoroatomskih molekula
  - Modelni hamiltonijan
  - Elektronske bazne funkcije
  - Elektronske bazne funkcije - dijabatizacija
  - Mala odstupanja od linearnosti
  - Ugao  $\tau$
  - S-O matični elementi
  - Vibronska baza
  - Rezultati
- 5 Model za molekule sa proizvoljnim brojem jezgara

- Molekul  $\implies$   $S$  jezgara ( $A, B, \dots, S$ )  
 $N$  elektrona ( $\alpha, \beta, \dots, N$ )

- Talasna funkcija

$$\Psi = \Psi(r_A, s_A, \dots, r_S, s_S, r_\alpha, s_\alpha, \dots, r_N, s_N)$$

- Šredingerova jednačina

$$\hat{H}\Psi = E\Psi$$

- Šredingerova jednačina

$$\hat{H}\Psi(\vec{r}, \vec{R}) = E\Psi(\vec{r}, \vec{R})$$

- Nerelativistički hamiltonijan molekula

$$\begin{aligned} \hat{H} = \hat{T}_n + \hat{T}_e + \hat{V}_{en} + \hat{V}_{ee} + \hat{V}_{nn} = & -\frac{1}{2} \sum_{K=A}^S \frac{1}{m_K} \Delta_K - \frac{1}{2} \sum_{\mu=\alpha}^N \Delta_\mu \\ & - \sum_{\mu=\alpha}^N \sum_{K=A}^S \frac{Z_K}{|\vec{r}_{\mu K}|} + \sum_{\mu=\alpha}^N \sum_{\nu>\mu}^N \frac{1}{|\vec{r}_{\mu\nu}|} + \sum_{K=A}^S \sum_{L>K}^S \frac{Z_K Z_L}{|\vec{R}_{KL}|} \end{aligned}$$

- Šredingerova jednačina

$$\hat{H}\Psi(\vec{r}, \vec{R}) = E\Psi(\vec{r}, \vec{R})$$

- Nerelativistički hamiltonijan molekula

$$\hat{H} = \cancel{\hat{T}_n} + \hat{T}_e + \hat{V}_{en} + \hat{V}_{ee} + \hat{V}_{nn} = \cancel{-\frac{1}{2} \sum_{K=A}^S \frac{1}{m_K} \Delta_K} - \frac{1}{2} \sum_{\mu=\alpha}^N \Delta_{\mu} \\ - \sum_{\mu=\alpha}^N \sum_{K=A}^S \frac{Z_K}{|\vec{r}_{\mu K}|} + \sum_{\mu=\alpha}^N \sum_{\nu>\mu}^N \frac{1}{|\vec{r}_{\mu\nu}|} + \sum_{K=A}^S \sum_{L>K}^S \frac{Z_K Z_L}{|\vec{R}_{KL}|}$$

- Elektronski hamiltonijan

$$\hat{H}_e = \hat{T}_e + \hat{V}_{en} + \hat{V}_{ee} + \hat{V}_{nn}$$

- Rešavamo elektronsku Šredingerovu jednačinu

$$\hat{H}_e \psi(\vec{r}; \vec{R}) = E_{el}(\vec{R}) \psi(\vec{r}; \vec{R})$$

- Rešavamo jezgarnu Šredingerovu jednačinu

$$\Psi(\vec{r}, \vec{R}) = \sum_j \Phi_j(\vec{R}) \psi_j(\vec{r}; \vec{R})$$

$$[\hat{T}_n + E_{el}(\vec{R}) - E] \Phi_j(\vec{R}) + \sum_j \Lambda_{ij} \Phi_j(\vec{R}) = 0$$

gde je

$$\Lambda_{ij} = -\frac{1}{2} \sum_{K=A}^S \frac{1}{m_K} \left[ 2 \langle \psi_i(\vec{r}; \vec{R}) | \vec{\nabla}_K \psi_j(\vec{r}; \vec{R}) \rangle \vec{\nabla}_K + \langle \psi_i(\vec{r}; \vec{R}) | \Delta \psi_j(\vec{r}; \vec{R}) \rangle \right]$$



- Zanemarivanje  $\Lambda_{ij}$ 
  - nedijagonalni članovi ( $i \neq j$ )  $\Rightarrow$  adijabatska aproksimacija
  - ( $i \neq j$ )  $\wedge$  ( $i = j$ )  $\Rightarrow$  B-O aproksimacija

- Dobijamo sistem jednačina

$$[\hat{T}_n + E_{el}(\vec{R})]\Phi_j(\vec{R}) = E\Phi_j(\vec{R}), \quad j = 1, 2, \dots$$

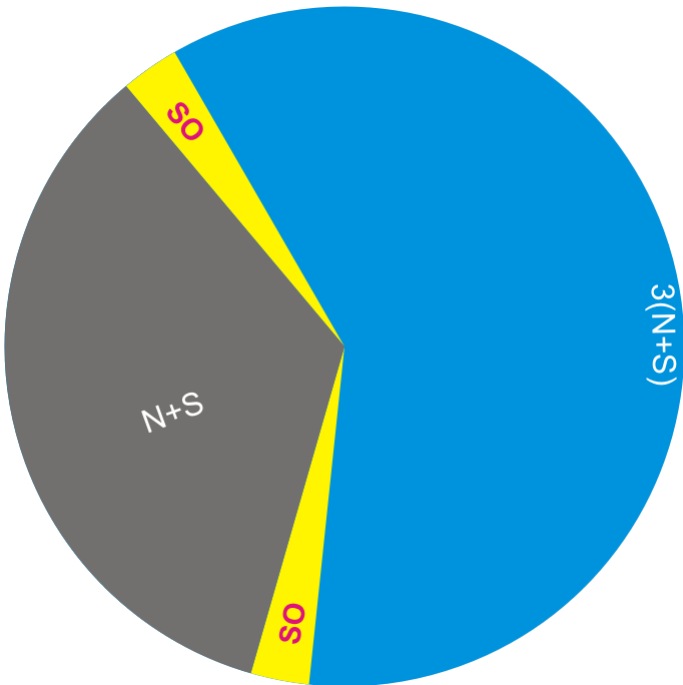
- Narušavanje B-O aproksimacije  $\implies$  **neadijabatski efekti**
  - i) izbegnuta presecanja potencijalnih površi
  - ii) Jan-Telerov efekat
  - iii) Rener-Telerov efekat

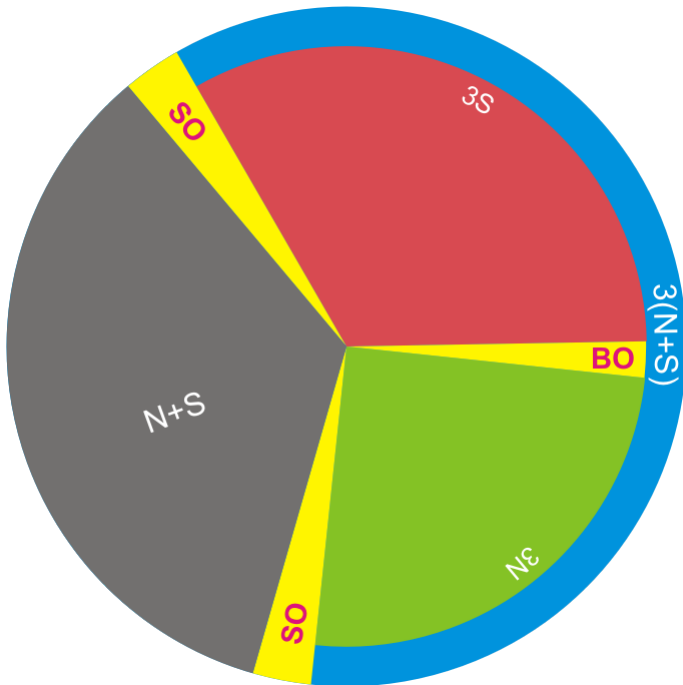
- "*Brute force*" metode
- Semiempirijske metode
- *Ab initio* metode

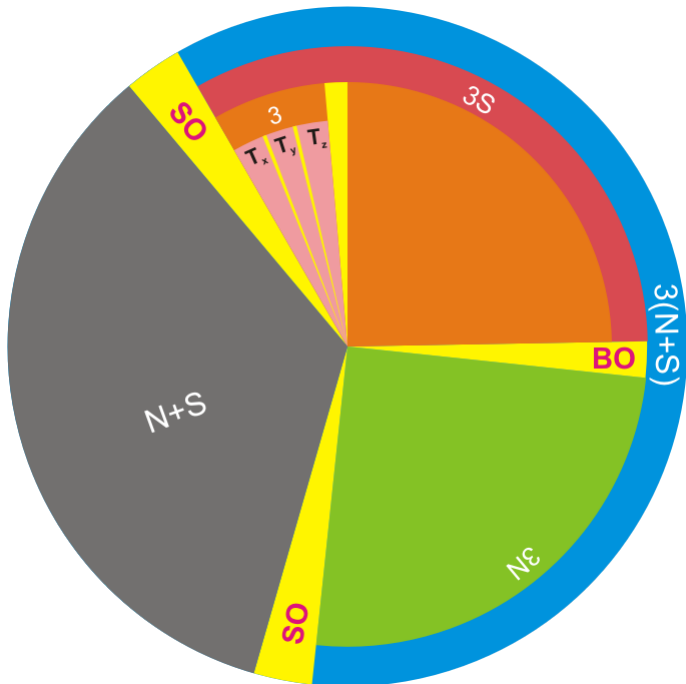
## Približne metode

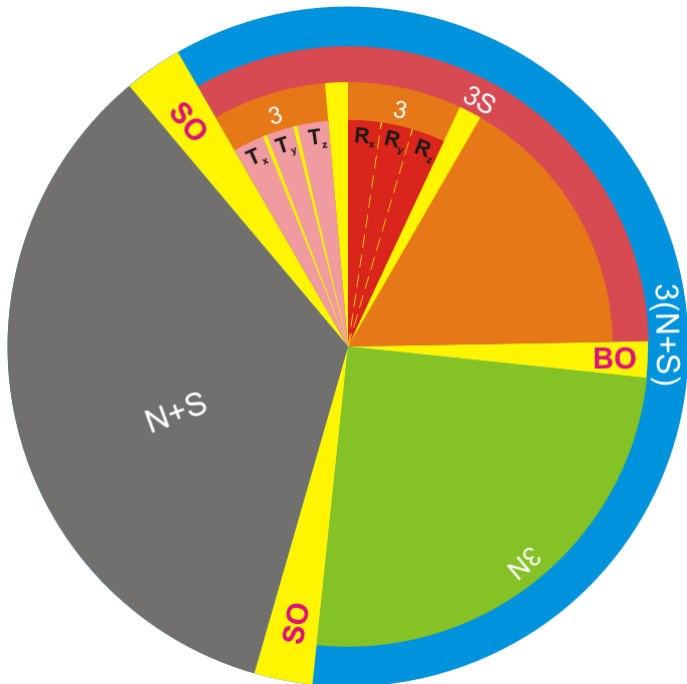
- Perturbacioni račun
- Varijacioni račun

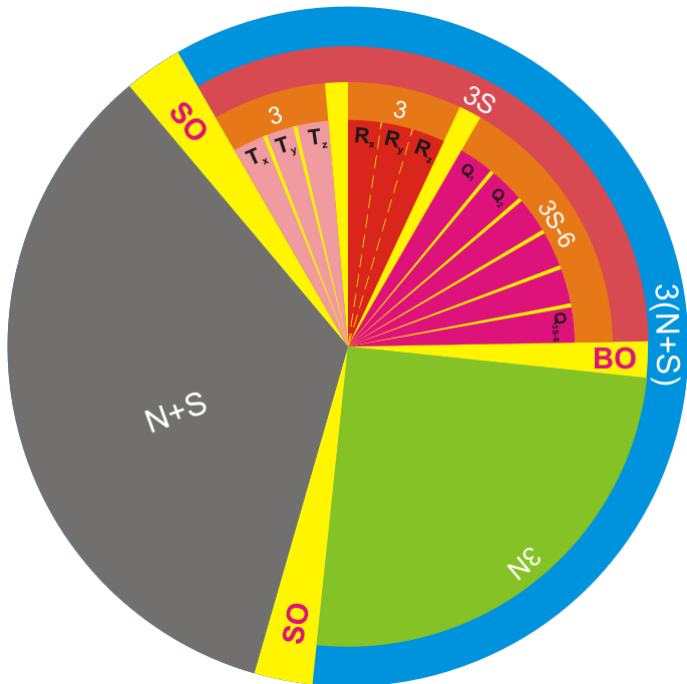
$$4(N+S)$$



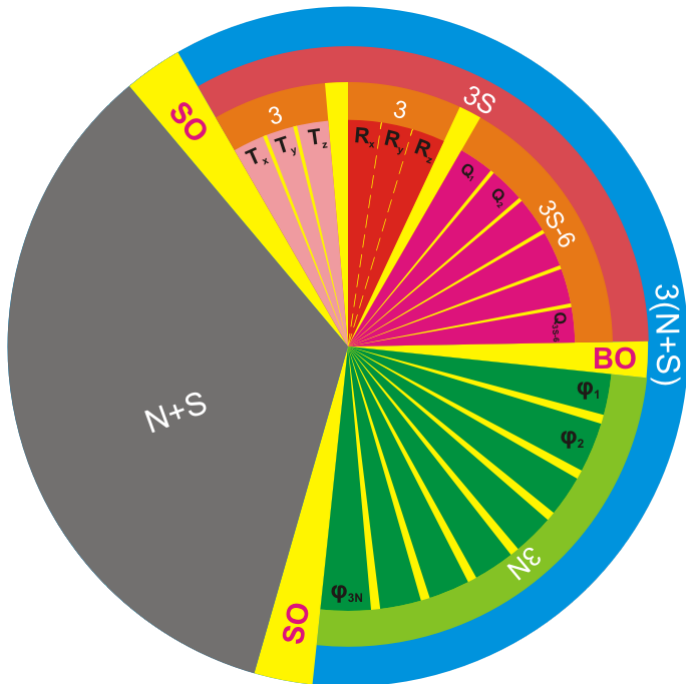












# Ab initio metode za rešavanje elektronskog problema

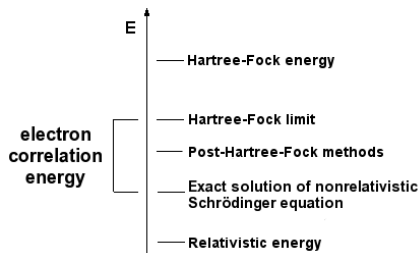
- Hartree-Fock method (HF)
- Density functional theory (DFT) (*ab initio?*)

## Post-Hartree-Fock methods

- Møller–Plesset perturbation theory ( $MP_n$ )
- Configuration interaction (CI)
- Coupled cluster (CC)

## Multi-reference methods

- Multi-configurational self-consistent field (MCSCF, CASSCF)
- Multi-reference configuration interaction (MRCI)
- Complete active space perturbation theory (CASPT $_n$ )
- Multi-reference coupled-cluster (MRCC)



# Sadržaj izlaganja

- 1 Kvantno-hemijski problem
  - Šredingerova jednačina
  - Born-Openhajmerova aproksimacija
  - *Ab initio* metode
- 2 Mali molekuli
- 3 Rener-TeleroV efekat - opšta razmatranja
- 4 Model za tretiranje R-T efekta kod četvoroatomskih molekula
  - Modelni hamiltonijan
  - Elektronske bazne funkcije
  - Elektronske bazne funkcije - dijabatizacija
  - Mala odstupanja od linearnosti
  - Ugao  $\tau$
  - S-O matični elementi
  - Vibronska baza
  - Rezultati
- 5 Model za molekule sa proizvoljnim brojem jezgara

	2 atoms	3 atoms	4 atoms	5 atoms	6 atoms	7 atoms	8 atoms	9 atoms	10 atoms	11 atoms	12 atoms	>12 atoms
H <sub>2</sub>	C <sub>3</sub> <sup>*</sup>	c-C <sub>3</sub> H	C <sub>5</sub> <sup>*</sup>	C <sub>9</sub> H	C <sub>6</sub> H	CH <sub>3</sub> C <sub>3</sub> N	CH <sub>3</sub> C <sub>4</sub> H	CH <sub>3</sub> C <sub>5</sub> N	HC <sub>9</sub> N	c-C <sub>6</sub> H <sub>6</sub> <sup>*</sup>	HC <sub>11</sub> N	
AlF	C <sub>2</sub> H	<i>f</i> -C <sub>3</sub> H	C <sub>4</sub> H	<i>f</i> -H <sub>2</sub> C <sub>4</sub>	CH <sub>2</sub> CHCN	HC(O)OCH <sub>3</sub>	CH <sub>3</sub> CH <sub>2</sub> CN	(CH <sub>3</sub> ) <sub>2</sub> CO	CH <sub>3</sub> C <sub>6</sub> H	<i>n</i> -C <sub>3</sub> H <sub>7</sub> CN	C <sub>60</sub> <sup>*</sup>	
AlCl	C <sub>2</sub> O	C <sub>3</sub> N	C <sub>4</sub> Si	C <sub>2</sub> H <sub>4</sub> <sup>*</sup>	CH <sub>3</sub> C <sub>2</sub> H	CH <sub>3</sub> COOH	(CH <sub>3</sub> ) <sub>2</sub> O	(CH <sub>2</sub> OH) <sub>2</sub>	C <sub>2</sub> H <sub>5</sub> OCHO	<i>f</i> -C <sub>3</sub> H <sub>7</sub> CN 2014	C <sub>70</sub> <sup>*</sup>	
C <sub>2</sub> <sup>**</sup>	C <sub>2</sub> S	C <sub>3</sub> O	<i>f</i> -C <sub>3</sub> H <sub>2</sub>	CH <sub>3</sub> CN	HC <sub>9</sub> N	C <sub>7</sub> H	CH <sub>3</sub> CH <sub>2</sub> OH	CH <sub>3</sub> CH <sub>2</sub> CHO	CH <sub>3</sub> OC(O)CH <sub>3</sub>	C <sub>2</sub> H <sub>5</sub> OCH <sub>3</sub> <sup>?</sup>	C <sub>60</sub> <sup>**</sup>	
CH	CH <sub>2</sub>	C <sub>3</sub> S	c-C <sub>3</sub> H <sub>2</sub>	CH <sub>3</sub> NC	CH <sub>3</sub> CHO	C <sub>6</sub> H <sub>2</sub>	HC <sub>7</sub> N					
CH <sup>+</sup>	HCN	C <sub>2</sub> H <sub>2</sub> <sup>*</sup>	H <sub>2</sub> CCN	CH <sub>3</sub> OH	CH <sub>3</sub> NH <sub>2</sub>	CH <sub>2</sub> OHCHO	C <sub>6</sub> H					
CN	HCO	NH <sub>3</sub>	CH <sub>4</sub> <sup>*</sup>	CH <sub>3</sub> SH	c-C <sub>2</sub> H <sub>4</sub> O	<i>f</i> -HC <sub>6</sub> H <sup>*</sup>	CH <sub>3</sub> C(O)NH <sub>2</sub>					
CO	HCO <sup>+</sup>	HCCN	HC <sub>3</sub> N	HC <sub>3</sub> NH <sup>*</sup>	H <sub>2</sub> CCHOH	CH <sub>2</sub> CHCHO (?)	C <sub>8</sub> H <sup>-</sup>					
CO <sup>+</sup>	HCS <sup>+</sup>	HCNH <sup>+</sup>	HC <sub>2</sub> NC	HC <sub>2</sub> CHO	C <sub>6</sub> H <sup>-</sup>	CH <sub>2</sub> CCHCN	C <sub>3</sub> H <sub>6</sub>					
CP	HOC <sup>+</sup>	HNCO	HCOOH	NH <sub>2</sub> CHO	CH <sub>3</sub> NCO 2015	H <sub>2</sub> NCH <sub>2</sub> CN	CH <sub>3</sub> CH <sub>2</sub> SH (?)					
SiC	H <sub>2</sub> O	HNCS	H <sub>2</sub> CNH	C <sub>9</sub> N		CH <sub>3</sub> CHNH						
HCl	H <sub>2</sub> S	HOCO <sup>+</sup>	H <sub>2</sub> C <sub>2</sub> O	<i>f</i> -HC <sub>4</sub> H <sup>*</sup>								
KCl	HNC	H <sub>2</sub> CO	H <sub>2</sub> CNCN	<i>f</i> -HC <sub>4</sub> N								
NH	HNO	H <sub>2</sub> CN	HNC <sub>3</sub>	c-H <sub>2</sub> C <sub>3</sub> O								
NO	MgCN	H <sub>2</sub> CS	SiH <sub>4</sub> <sup>*</sup>	H <sub>2</sub> C <sub>2</sub> NH (?)								
NS	MgNC	H <sub>3</sub> O <sup>+</sup>	H <sub>2</sub> COH <sup>+</sup>	C <sub>5</sub> N <sup>-</sup>								
NaCl	N <sub>2</sub> H <sup>+</sup>	c-SiC <sub>3</sub>	C <sub>4</sub> H <sup>-</sup>	HNCHCN								
OH	N <sub>2</sub> O	CH <sub>3</sub> <sup>*</sup>	HC(O)CN									
PN	NaCN	C <sub>3</sub> H <sup>-</sup>	HNCNH									
SO	OCS	PH <sub>3</sub>	CH <sub>3</sub> O									
SO <sup>+</sup>	SO <sub>2</sub>	HCNO	NH <sub>4</sub> <sup>+</sup>									
SiN	c-SiC <sub>2</sub>	HOCN	H <sub>2</sub> NCO <sup>+</sup> (?)									
SiO	CO <sub>2</sub> <sup>*</sup>	HSCN	NCCNH <sup>+</sup> 2015									

## Extragalactic Molecules (as of 10/2015)

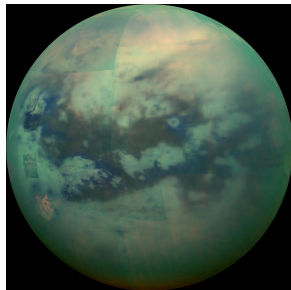
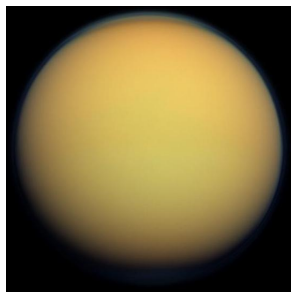
	2 atoms	3 atoms	4 atoms	5 atoms	6 atoms	7 atoms	8 atoms	>8 atoms
	OH	H <sub>2</sub> O	H <sub>2</sub> CO	<i>c</i> -C <sub>3</sub> H <sub>2</sub>	CH <sub>3</sub> OH	CH <sub>3</sub> CCH	HC <sub>6</sub> H	<i>c</i> -C <sub>6</sub> H <sub>6</sub> *
	CO	HCN	NH <sub>3</sub>	HC <sub>3</sub> N	CH <sub>3</sub> CN	CH <sub>3</sub> NH <sub>2</sub>		C <sub>60</sub> * (?)
	H <sub>2</sub> *	HCO <sup>+</sup>	HNCO	CH <sub>2</sub> NH	HC <sub>4</sub> H *	CH <sub>3</sub> CHO		
	CH	C <sub>2</sub> H	C <sub>2</sub> H <sub>2</sub> *	NH <sub>2</sub> CN	HC(O)NH <sub>2</sub>			
	CS	HNC	H <sub>2</sub> CS ?	<i>f</i> -C <sub>3</sub> H <sub>2</sub>				
	CH <sup>+</sup> **	N <sub>2</sub> H <sup>+</sup>	HOCO <sup>+</sup>	H <sub>2</sub> CCN				
	CN	OCS	<i>c</i> -C <sub>3</sub> H	H <sub>2</sub> CCO				
	SO	HCO	H <sub>3</sub> O <sup>+</sup>	C <sub>4</sub> H				
	SiO	H <sub>2</sub> S	<i>f</i> -C <sub>3</sub> H					
	CO <sup>+</sup>	SO <sub>2</sub>						
	NO	HOC <sup>+</sup>						
	NS	C <sub>2</sub> S						
	NH	H <sub>2</sub> O <sup>+</sup>						
	OH <sup>+</sup>	HCS <sup>+</sup>						
	HF	H <sub>2</sub> Cl <sup>+</sup> 2014						
	SO <sup>+</sup>	NH <sub>2</sub> 2014						
	AlH <sup>+</sup> 2015							

# Mali molekuli - atmosfera Titana

$\text{N}\equiv\text{C}-\text{C}\equiv\text{C}-\text{C}\equiv\text{N}$  Dcijanoacetilen ( $\text{C}_4\text{N}_2$ )

$\text{H}-\text{C}\equiv\text{C}-\text{C}\equiv\text{C}-\text{H}$  Diacetilen ( $\text{C}_4\text{H}_2$ )

- Cijanoacetilen i cijanodiacetilen opaženi u interstelarnom prostoru, dicijanoacetilen?
- Atmosfera Titana ( $\text{N}_2$ ,  $\text{CH}_4$ , ...)
- $\text{C}_4\text{N}_{2(s)}$  detektovan
- $\text{C}_4\text{N}_2$  značajan za meteorologiju Titana
- $X\ ^2\Pi_u$  elektronsko stanje katjona  $\text{C}_4\text{N}_2^+$ , ispoljava **Rener-Telerov efekat**

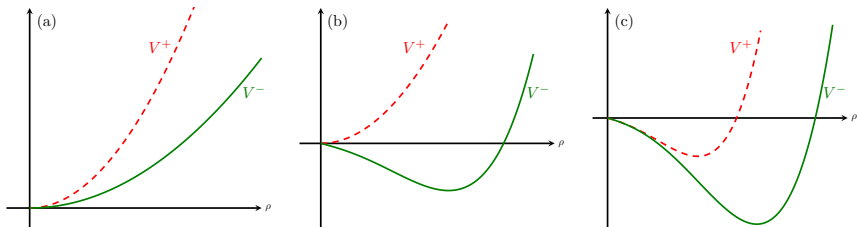


# Sadržaj izlaganja

- 1 Kvantno-hemijski problem
  - Šredingerova jednačina
  - Born-Openhajmerova aproksimacija
  - *Ab initio* metode
- 2 Mali molekuli
- 3 Rener-Telerov efekat - opšta razmatranja
- 4 Model za tretiranje R-T efekta kod četvoroatomskih molekula
  - Modelni hamiltonijan
  - Elektronske bazne funkcije
  - Elektronske bazne funkcije - dijabatizacija
  - Mala odstupanja od linearnosti
  - Ugao  $\tau$
  - S-O matični elementi
  - Vibronska baza
  - Rezultati
- 5 Model za molekule sa proizvoljnim brojem jezgara

# Rener-Telerov efekat - opšta razmatranja

- Linearni molekuli (elektronska stanja  $\Pi$ ,  $\Delta$ ,  $\Phi$ , ...)
- $C_{\infty v}$ ,  $D_{\infty h} \rightarrow C_S$ ,  $C_{2v}$ ,  $C_{2h}$ , ...
- Vibraciono-elektronska (vibronska) sprega



- Troatomske molekuli
- Četvoroatomske molekuli



# Sadržaj izlaganja

- 1 Kvantno-hemijski problem
  - Šredingerova jednačina
  - Born-Openhajmerova aproksimacija
  - *Ab initio* metode
- 2 Mali molekuli
- 3 Rener-Telerov efekat - opšta razmatranja
- 4 Model za tretiranje R-T efekta kod četvoroatomskih molekula
  - Modelni hamiltonijan
  - Elektronske bazne funkcije
  - Elektronske bazne funkcije - dijabatizacija
  - Mala odstupanja od linearnosti
  - Ugao  $\tau$
  - S-O matrični elementi
  - Vibronska baza
  - Rezultati
- 5 Model za molekule sa proizvoljnim brojem jezgara

- a) Razmatrano elektronsko stanje je dovoljno odvojeno od drugih elektronskih stanja
- b) Ravnotežna geometrija molekula je linearna
- c) Harmonijska aproksimacija
- d) Sprega između savijajućih i istežućih vibracija je zanemarena
- e) Rotacija molekula u celini zanemarena
- f) Korišćene krivolinijske unutrašnje koordinate
- g) Primenjen operator kinetičke energije jezgara za infinitezimalne savijajuće vibracije
- h) Primenjen fenomenološki spin-orbitni operator
- i) Asimptotske (linearne) elektronske talasne funkcije su korišćene za matičnu reprezentaciju operatora kinetičke energije jezgara

- Molekulski hamiltonijan

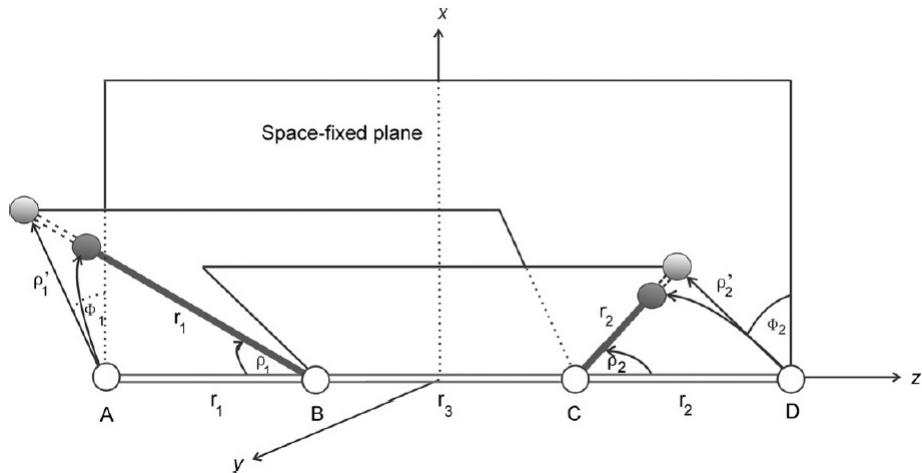
$$\hat{H} = \hat{H}_e + \hat{T}_n + \hat{H}_{SO}$$

- Elektronski spin-orbitni operator

$$\hat{H}_{SO} = A_{SO} \hat{L}_z \hat{S}_z$$

- Operator kinetičke energije za savijajuća kretanja jezgara
  - i) krivolinijske unutrašnje koordinate  $(\rho_1, \phi_1, \rho_2, \phi_2)$
  - ii) krivolinijske simetrijske koordinate  $(\rho_C, \phi_C, \rho_T, \phi_T)$
  - iii) pravolinijske unutrašnje/simetrijske koordinate
  - iv) normalne koordinate

# Modelni hamiltonijan



[Perić *et al.* 2006]

- $\hat{T}_n$  u krivolinijskim unutrašnjim koordinatama za dve (dvostruko degenerisane) infinitezimalne savijajuće vibracije

$$\begin{aligned}\hat{T}_n = & -\frac{1}{2\mu_1} \left( \frac{\partial^2}{\partial \rho_1^2} + \frac{1}{\rho_1} \frac{\partial}{\partial \rho_1} + \frac{1}{\rho_1^2} \frac{\partial^2}{\partial \phi_1^2} \right) - \frac{1}{2\mu_2} \left( \frac{\partial^2}{\partial \rho_2^2} + \frac{1}{\rho_2} \frac{\partial}{\partial \rho_2} + \frac{1}{\rho_2^2} \frac{\partial^2}{\partial \phi_2^2} \right) \\ & + \frac{1}{\mu_{12}} \left[ \cos(\phi_2 - \phi_1) \left( \frac{\partial^2}{\partial \rho_1 \partial \rho_2} + \frac{1}{\rho_1 \rho_2} \frac{\partial^2}{\partial \phi_1 \partial \phi_2} \right) \right. \\ & \left. + \sin(\phi_2 - \phi_1) \left( \frac{1}{\rho_1} \frac{\partial^2}{\partial \rho_2 \partial \phi_1} - \frac{1}{\rho_2} \frac{\partial^2}{\partial \rho_1 \partial \phi_2} \right) \right]\end{aligned}$$

$$V = \frac{1}{2} k_1 \rho_1^2 + \frac{1}{2} k_2 \rho_2^2 + k_{12} \rho_1 \rho_2 \cos(\phi_2 - \phi_1)$$

- Vibronska talasna funkcija u obliku

$$\Psi = f_1\psi_1 + f_2\psi_2$$

- Izbor elektronskih talasnih funkcija  $\Rightarrow$  adijabatske talasne funkcije

$$\hat{H}_e\psi^+ = V^+\psi^+, \quad \hat{H}_e\psi^- = V^-\psi^-$$

- Matrični elementi od  $\hat{H}_{el}$  dijagonalni
- Matrični elementi od  $\hat{T}_n$  sadrže članove:

$$\langle \psi^+ | \frac{\partial}{\partial q_i} | \psi^+ \rangle = \langle \psi^- | \frac{\partial}{\partial q_i} | \psi^- \rangle = \frac{\partial}{\partial q_i}$$

$$\langle \psi^+ | \frac{\partial}{\partial q_i} | \psi^- \rangle = \left\langle \psi^+ \left| \frac{\partial \psi^-}{\partial q_i} \right. \right\rangle = - \langle \psi^- | \frac{\partial}{\partial q_i} | \psi^+ \rangle$$

- B-O aproksimacija?
- Izbegnuta presecanja; članovi  $\rho_i^{-1}$  i  $\rho_i^{-1}\rho_j^{-1}$

- Dijabatske elektronske talasne funkcije

$$\eta_1 = \psi^+ \cos(\Lambda\tau) - \psi^- \sin(\Lambda\tau)$$

$$\eta_2 = \psi^+ \sin(\Lambda\tau) + \psi^- \cos(\Lambda\tau)$$

- Matrični elementi elektronskog operatora u ovoj bazi

$$E_{11} = \langle \eta_1 | \hat{H}_e | \eta_1 \rangle = V^+ \cos^2(\Lambda\tau) + V^- \sin^2(\Lambda\tau)$$

$$E_{22} = \langle \eta_2 | \hat{H}_e | \eta_2 \rangle = V^+ \sin^2(\Lambda\tau) + V^- \cos^2(\Lambda\tau)$$

$$E_{12} = \langle \eta_1 | \hat{H}_e | \eta_2 \rangle = \frac{V^+ - V^-}{2} \sin(2\Lambda\tau) = \langle \eta_2 | \hat{H}_e | \eta_1 \rangle$$

- Ugao  $\tau$  određujemo kao

$$\left\langle \eta_1 \left| \frac{\partial \eta_2}{\partial q_i} \right. \right\rangle = 0, \quad \frac{\partial \tau}{\partial q_i} = -\frac{1}{\Lambda} \left\langle \psi^+ \left| \frac{\partial \psi^-}{\partial q_i} \right. \right\rangle = \frac{1}{\Lambda} \left\langle \psi^- \left| \frac{\partial \psi^+}{\partial q_i} \right. \right\rangle$$

- Sledeća transformacija - linearne bazne funkcije

$$\psi_1 = \frac{1}{\sqrt{2}}(\eta_1 + i\eta_2) = \frac{1}{\sqrt{2}}e^{i\Lambda\tau}(\psi^+ + i\psi^-)$$

$$\psi_2 = \frac{1}{\sqrt{2}}(\eta_1 - i\eta_2) = \frac{1}{\sqrt{2}}e^{-i\Lambda\tau}(\psi^+ - i\psi^-)$$

- U linearnoj bazi matični elementi elektronskog operatora oblika

$$E_{11}^{lin} = \langle \psi_1 | \hat{H}_e | \psi_1 \rangle = \frac{V^+ + V^-}{2} = \langle \psi_2 | \hat{H}_e | \psi_2 \rangle = E_{22}^{lin}$$

$$E_{12}^{lim} = \langle \psi_1 | \hat{H}_e | \psi_2 \rangle = \frac{V^+ - V^-}{2}e^{-2i\tau}$$

$$E_{21}^{lim} = \langle \psi_2 | \hat{H}_e | \psi_1 \rangle = \frac{V^+ - V^-}{2}e^{2i\tau}$$

- Kompleksni matični elementi "linearnih" el. funkcija po koordinatama jezgara



- Asimptotske forme adijabatskih el. talasnih funkcija

$$\psi_0^+ = \lim_{\rho_1 \rightarrow 0, \rho_2 \rightarrow 0} \psi^+ = \xi \frac{1}{\sqrt{\pi}} \cos[\Lambda(\theta - \tau)]$$

$$\psi_0^- = \lim_{\rho_1 \rightarrow 0, \rho_2 \rightarrow 0} \psi^- = \xi \frac{1}{\sqrt{\pi}} \sin[\Lambda(\theta - \tau)]$$

- Dijabatske elektronske funkcije

$$\eta_0^1 = \lim_{\rho_1 \rightarrow 0, \rho_2 \rightarrow 0} \eta_1 = \xi \frac{1}{\sqrt{\pi}} \cos(\Lambda\theta)$$

$$\eta_0^2 = \lim_{\rho_1 \rightarrow 0, \rho_2 \rightarrow 0} \eta_2 = \xi \frac{1}{\sqrt{\pi}} \sin(\Lambda\theta)$$

- "Linearne" elektronske funkcije

$$\psi_0^1 = \lim_{\rho_1 \rightarrow 0, \rho_2 \rightarrow 0} \psi_1 = \xi \frac{1}{\sqrt{2\pi}} e^{i\Lambda\theta}$$

$$\psi_0^2 = \lim_{\rho_1 \rightarrow 0, \rho_2 \rightarrow 0} \psi_2 = \xi \frac{1}{\sqrt{2\pi}} e^{-i\Lambda\theta}$$

- U okviru harm. aproksimacije i za  $\Pi$  elektronsko stanje, sledi

$$\tau = -\frac{1}{2} \arctan \left\{ \frac{\epsilon_1 \rho_1^2 \sin(2\phi_1) + \epsilon_1 \rho_2^2 \sin(2\phi_2) + 2\epsilon_{12} \rho_1 \rho_2 \sin(\phi_1 + \phi_2)}{\epsilon_1 \rho_1^2 \cos(2\phi_1) + \epsilon_1 \rho_2^2 \cos(2\phi_2) + 2\epsilon_{12} \rho_1 \rho_2 \cos(\phi_1 + \phi_2)} \right\}$$

- Izvodi

$$\frac{\partial \tau}{\partial \rho_2} = \frac{\rho_1 \sin(\phi_2 - \phi_1) [\epsilon_1 \epsilon_{12} \rho_1^2 + \epsilon_2 \epsilon_{12} \rho_2^2 + 2\epsilon_1 \epsilon_2 \rho_1 \rho_2 \cos(\phi_2 - \phi_1)]}{(V^+ - V^-)^2}$$

$$\frac{\partial \tau}{\partial \phi_2} = \frac{\epsilon_2^2 \rho_2^4 + 2\epsilon_{12}^2 \rho_1^2 \rho_2^2 + \epsilon_{12} \rho_1 \rho_2 (\epsilon_1 \rho_1^2 + \epsilon_2 \rho_2^2) \cos(\phi_2 - \phi_1) + \epsilon_1 \epsilon_2 \rho_1^2 \rho_2^2 \cos 2(\phi_2 - \phi_1)}{(V^+ - V^-)^2}$$

- Matični elementi operatora S-O sprege u "linearnoj" asimptotskoj bazi

$$\langle \psi_0^1 | \hat{H}_{SO} | \psi_0^1 \rangle = \Lambda \Sigma A_{SO}$$

$$\langle \psi_0^2 | \hat{H}_{SO} | \psi_0^2 \rangle = -\Lambda \Sigma A_{SO}$$

$$\langle \psi_0^1 | \hat{H}_{SO} | \psi_0^2 \rangle = \langle \psi_0^2 | \hat{H}_{SO} | \psi_0^1 \rangle = 0$$

- Elektronski matični elementi su operatori koji treba da deluju na vibracione bazne funkcije
- Svojsvene funkcije 2D harmonijskog oscilatora

$$\hat{N}_z = \hat{R}_z + \hat{L}_z \quad \Rightarrow \quad K = l \pm \Lambda$$

$$\hat{J}_z = \hat{N}_z + \hat{S}_z \quad \Rightarrow \quad P = K + \Sigma$$

$$l = v, v - 2, \dots, 1 \quad \text{ili} \quad 0$$

- Adijabatske energije  $\Rightarrow$  FV-SA-CASSCF(9,10)/cc-pVQZ

- Dužine veza konstantne

$$\text{C-H} \equiv r = 2,04 \text{ bohr} = 1,0795 \text{ \AA}$$

$$\text{C-C} \equiv R = 2,37 \text{ bohr} = 1,2542 \text{ \AA}$$

- MOLPRO 2012.1 [[Werner et al. 2012](#)]

# Parametri koji ulaze u model

TABLE I. Adiabatic electronic energies ( $V^+, V^-$ ) for the components of the  $X^2\Pi_u$  electronic state of  $C_2H_2^+$  derived from CASSCF calculations at the C-H and C-C bond lengths kept fixed at 2.04 and 2.37 bohrs, respectively.  $\bar{V} = (V^+ + V^-)/2$ ,  $\Delta V = (V^+ - V^-)$ ,  $V_e$  is the energy at linear (equilibrium) geometry (computed in the  $D_{2h}$  point group).

$\rho_1$ (deg)	$\phi_1$ (deg)	$\rho_2$ (deg)	$\phi_2$ (deg)	$V^+$ (hartree)	$V^-$ (hartree)	$\bar{V} - V_e$ (hartree)	$\Delta V$ (hartree)
0	0	0	0	-76.621 734 28	-76.621 734 28	0.0	0.0
10	0	9.99	0.01	-76.619 367 82	-76.619 272 02	0.002 414 36	-0.000 095 80
10	0	9.99	179.99	-76.619 846 90	-76.620 680 52	0.001 470 57	0.000 833 62

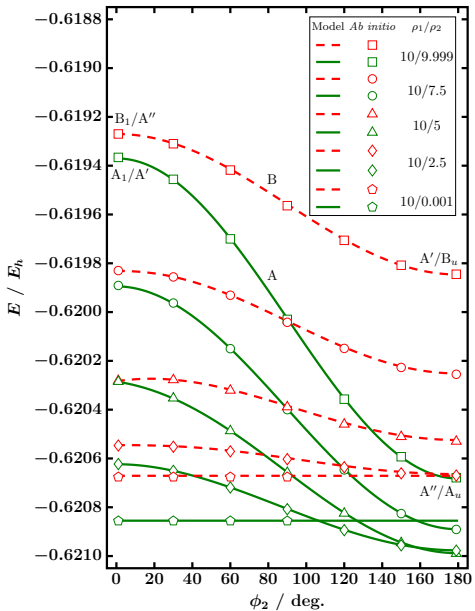
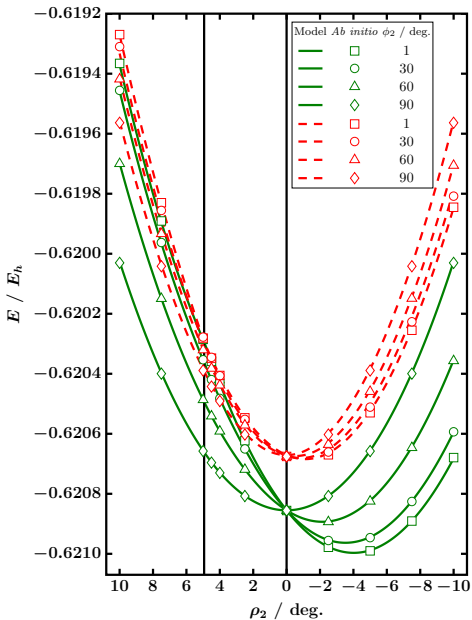
TABLE II. Molecular parameters for the  $X^2\Pi_u$  electronic state of  $C_2H_2^+$  derived from CASSCF calculations. All quantities are given in atomic units (hartree/rad<sup>2</sup>) except of  $u_1, u_2$ , which are dimensionless.

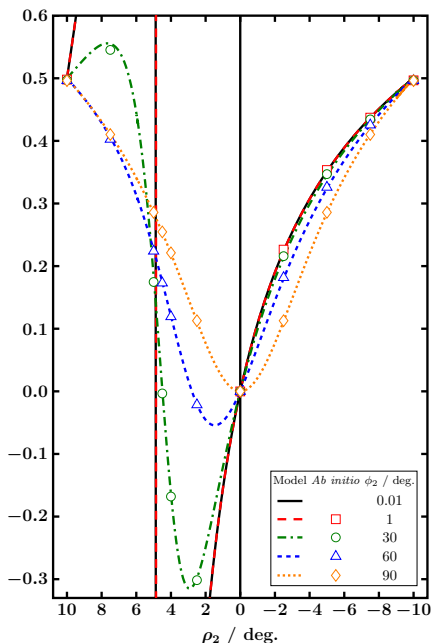
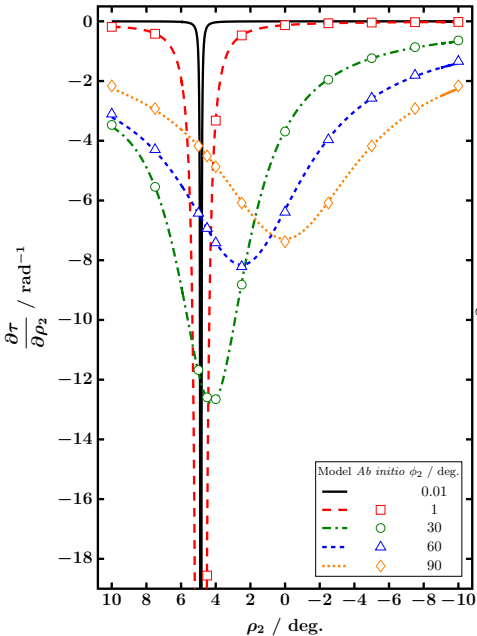
$k_T = 0.096\,542$	$k_C = 0.158\,508$	$\varepsilon_T = 0.027\,366$	$\varepsilon_C = -0.003\,145$
$k_1 = k_2 = 0.063\,763$	$k_{12} = 0.015\,492$	$\varepsilon_1 = \varepsilon_2 = 0.006\,055$	$\varepsilon_{12} = -0.007\,628$
$u_1 = 2.025\,715$	$u_2 = 0.493\,653$		

- Presecanja pri planarnim geometrijama [Perić 2006]

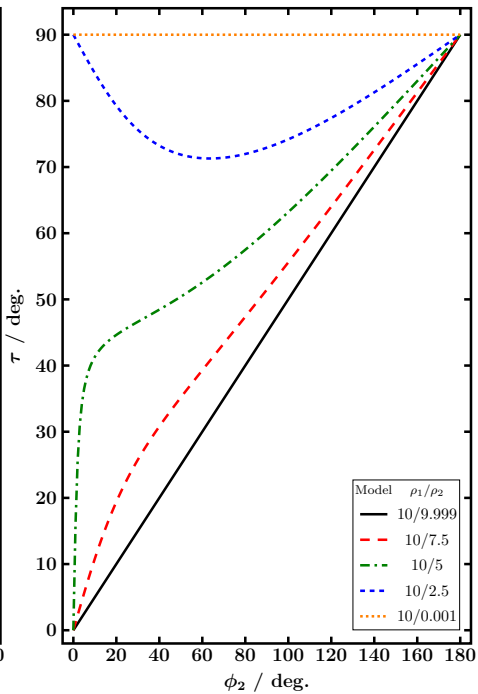
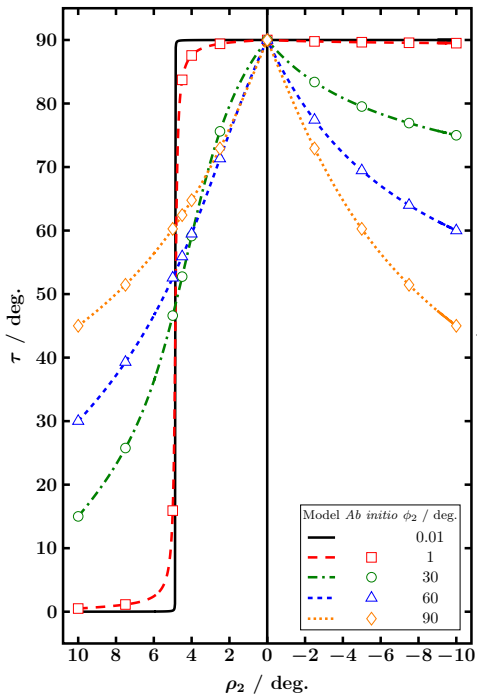
$$\rho_2 = u\rho_1, \quad u = \frac{-\varepsilon_{12} \pm \sqrt{\varepsilon_{12}^2 - \varepsilon_1\varepsilon_2}}{\varepsilon_2}$$

- Ukupno 5 *ab initio* računa!









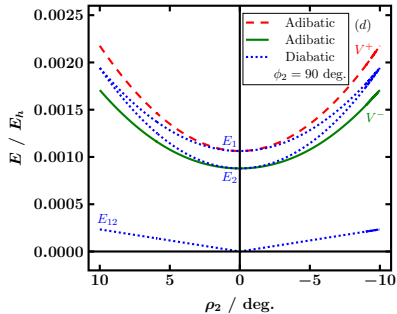
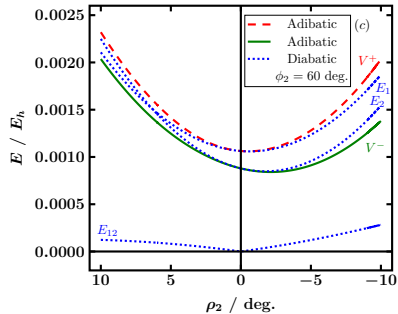
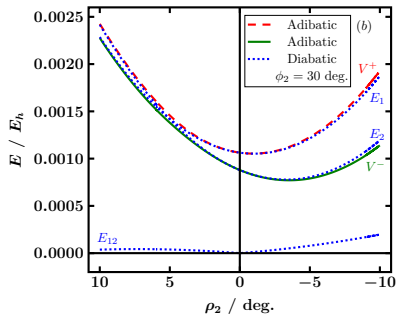
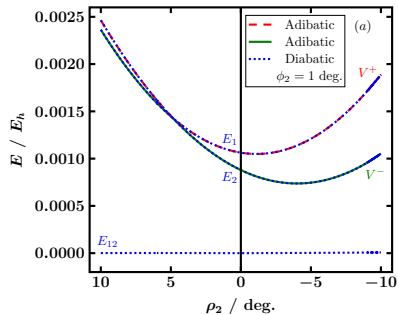


TABLE III. Low-lying vibronic levels (in  $cm^{-1}$ ) of the  $X^2\Pi_u$  electronic state of  $C_2H_2^+$ . The set of parameters used in calculations of entries from columns Pert. and Var. is  $\omega_T \equiv \omega_4 = 692.32 \text{ cm}^{-1}$ ,  $\omega_C \equiv \omega_5 = 704.05 \text{ cm}^{-1}$ ,  $e_4 = 0.2923$ ,  $e_5 = 0.0219$  (the parameters  $e_4, e_5$  are dimensionless), and  $A_{SO} = -31.42 \text{ cm}^{-1}$ . The numbers in parentheses are differences between the values from the corresponding column and those from column Expt.

$v_T, v_C$	State	Expt. <sup>a</sup>	Pert. <sup>b</sup>	Var. <sup>b</sup>
0,0	$\Pi_{3/2u}$	0	0(0)	0(0)
	$\Pi_{1/2u}$	29.8	31.4(+2)	30.7(+1)
1,0	$\Sigma_u^-$	499.5	505.1(+6)	502.7(+3)
	$\Sigma_u^+$	501.7	505.1(+3)	502.7(+1)
	$\Delta_{5/2u}$	672.9	662.7(-10)	662.3(-10)
	$\Delta_{3/2u}$	701.4	694.2(-7)	691.6(-10)
	$\Sigma_u^+$	909.9	911.0(+1)	913.0(+3)
	$\Sigma_u^-$	912.1	911.0(-1)	913.0(+1)
0,1	$\Sigma_g^+$	694.9	697.7(+3)	697.8(+3)
	$\Sigma_g^-$	696.5	697.7(+1)	697.8(+1)
	$\Delta_{5/2g}$	713.4	703.9(-9)	703.9(-9)
	$\Delta_{3/2g}$	743.0	735.3(-8)	734.6(-8)
	$\Sigma_g^-$	738.2	741.8(+4)	741.0(+3)
	$\Sigma_g^+$	739.8	741.8(+2)	741.0(+1)
2,0	$\Pi_{3/2u}$	1108.3	1105.1(-3)	1102.2(-6)
	$\Pi_{1/2u}$	1110.0	1107.6(-2)	1104.5(-5)
	$\Phi_{7/2u}$	1316.0	1310.7(-5)	1310.4(-6)
	$\Phi_{5/2u}$	1342.7	1342.1(-1)	1337.9(-5)
	$\Pi_{1/2u}$	1683.5	1678.4(-5)	1679.3(-4)
	$\Pi_{3/2u}$	1685.3	1680.8(-4)	1682.6(-3)
1,1	$\Pi_{3/2g}$	1210.2	1208.3(-2)	1206.0(-4)
	$\Pi_{1/2g}$	1209.9	1208.4(-1)	1206.0(-4)
	$\Phi_{7/2g}$	1370.4	1366.6(-4)	1366.2(-4)
	$\Phi_{5/2g}$	1398.9	1398.0(-1)	1395.5(-3)
	$\Pi_{3/2g}$	1373.1	1367.0(-6)	1366.5(-7)
	$\Pi_{1/2g}$	1401.6	1398.4(-3)	1395.8(-5)
	$\Pi_{1/2g}$	1620.6	1615.5(-5)	1617.4(-4)
	$\Pi_{3/2g}$	1620.4	1615.5(-5)	1617.4(-3)
	$\Pi_{3/2u}$	1404.8	1396.8(-8)	1396.9(-8)
0,2	$\Pi_{1/2u}$	1399.9	1397.0(-3)	1397.0(-3)
	$\Phi_{7/2u}$	1410.7	1407.7(-3)	1407.7(-3)
	$\Phi_{5/2u}$	1440.5	1439.1(-1)	1438.4(-2)
	$\Pi_{1/2u}$	1451.2	1450.6(-1)	1449.8(-1)
	$\Pi_{3/2u}$	1449.3	1450.7(+1)	1449.9(+1)

# Sadržaj izlaganja

- 1 Kvantno-hemijski problem
  - Šredingerova jednačina
  - Born-Openhajmerova aproksimacija
  - *Ab initio* metode
- 2 Mali molekuli
- 3 Rener-TeleroV efekat - opšta razmatranja
- 4 Model za tretiranje R-T efekta kod četvoroatomskih molekula
  - Modelni hamiltonijan
  - Elektronske bazne funkcije
  - Elektronske bazne funkcije - dijabatizacija
  - Mala odstupanja od linearnosti
  - Ugao  $\tau$
  - S-O matični elementi
  - Vibronska baza
  - Rezultati
- 5 Model za molekule sa proizvoljnim brojem jezgara

# Model za molekule sa proizvoljnim brojem jezgara

$$\begin{aligned}\hat{H} &= \hat{H}_e + \hat{T}_n + \hat{H}_{SO} \\ &= \hat{H}_e - \frac{1}{2} \sum_{i=1}^{S-2} \left( \frac{\partial^2}{\partial q_i^2} + \frac{1}{q_i} \frac{\partial}{\partial q_i} + \frac{1}{q_i^2} \frac{\partial^2}{\partial \phi_i^2} \right) \omega_i + A_{SO} \hat{L}_z \hat{S}_z\end{aligned}$$

$$\tau = -\frac{1}{2} \arctan \left\{ \frac{\sum_{i=1}^{S-2} \sum_{j=1}^{S-2} \varepsilon_{ij} \sqrt{\omega_i \omega_j} q_i q_j \sin(\phi_i + \phi_j)}{\sum_{i=1}^{S-2} \sum_{j=1}^{S-2} \varepsilon_{ij} \sqrt{\omega_i \omega_j} q_i q_j \sin(\phi_i + \phi_j)} \right\}$$

- Izbegnuta presecanja i pri planarnim i pri neplanarnim geometrijama
- Model testiran na primeru  $X^2\Pi_u$  stanja molekula  $C_5^-$

# Vibronski spektar $C_5^-$

**Table 1**  
Low-lying  $K = 0$  and  $K = 2$  vibronic energy levels (in  $\text{cm}^{-1}$ ) in the  $X^2\Pi_u$  electronic state of  $C_5^-$ .

$v_T, v_{C1}, v_{C2}$	$E^{(0)}$	$K = 0$				$K = 2$			
		Pert. <sup>a</sup>		Var. <sup>b</sup>		Pert. <sup>a</sup>		Var. <sup>b</sup>	
		$\Sigma = -1/2$	$\Sigma = 1/2$	$\Sigma = -1/2$	$\Sigma = 1/2$	$\Sigma = -1/2$	$\Sigma = 1/2$	$\Sigma = -1/2$	$\Sigma = 1/2$
0,0,1	1171	1098 1123	1098 1123	1091 1117	1091 1117	1112	1134	1108	1127
1,0,0	1347	1219 1409	1219 1409	1218 1402	1218 1402	1290	1312	1288	1307
0,0,3	1487	1369 1399	1369 1399	1358 1396	1358 1396	1380 1389	1360 1408	1364 1392	1356 1402
0,1,0	1534	1373 1684	1373 1684	1385 1677	1385 1677	1452	1474	1446	1464
1,0,2	1663			1493 1547 1561 1663	1493 1547 1561 1663	1495 1564 1682	1491 1586 1686	1493 1559 1659	1492 1575 1664
0,0,5	1803	1639 1677	1639 1677	1632 1677	1632 1677	1647 1669	1632 1684	1634 1674	1630 1681
0,1,2	1850		2051	1660 1670 1704 2051	1660 1670 1704 2051	1690 1684 1998	1687 1706 2002	1681 1704 2039	1692 1705 2037
1,0,4	1979		1770	1770 1820 1843 1939	1770 1820 1843 1939		1770	1770 1829 1839 1929	
3,0,0	2015	1786 2165	1786 2165	1785 2147	1785 2147	1809 2141	1813 2138	1809 2132	1811 2138
1,1,1	2026	1820 <sup>c</sup> 1820 <sup>c</sup> 1926 <sup>c</sup> 1926 <sup>c</sup> 2182 <sup>c</sup> 2182 <sup>c</sup>	1813 1919 1930 2064 2173 2194	1813 1919 1930 2064 2173 2194	1844 <sup>d</sup> 2015 <sup>d</sup> 2015 <sup>d</sup> 2208 <sup>d</sup>	1844 <sup>d</sup> 2037 <sup>d</sup> 2037 <sup>d</sup> 2208 <sup>d</sup>	1821 1870 1945 2175	1828 1882 1954 2179	

# Vibronski spektar $C_5^-$

**Table 2**

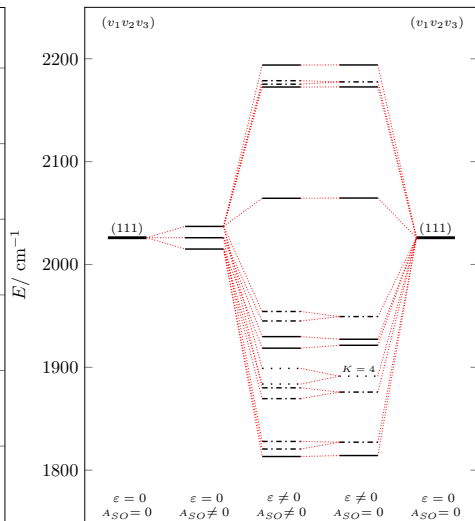
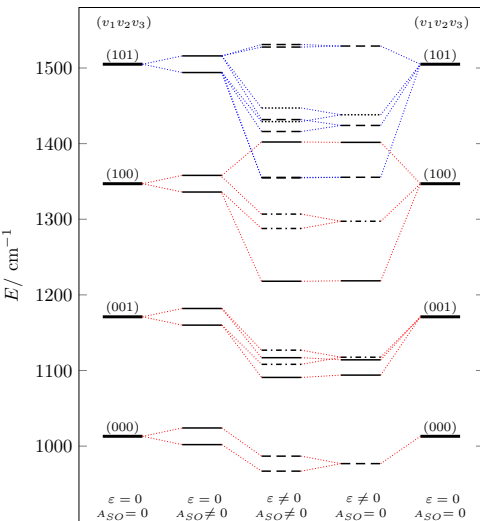
Low-lying  $K = 1$  and  $K = 3$  vibronic energy levels in the  $X^2I_0$  electronic state of  $C_5^-$ .

$v_T, v_{C1}, v_{C2}$	$E^{(0)}$	$K = 1$				$K = 3$							
		Pert. <sup>a</sup>		Var. <sup>b</sup>		Pert. <sup>a</sup>		Var. <sup>b</sup>					
		$\Sigma = -1/2$	$\Sigma = 1/2$	$\Sigma = -1/2$	$\Sigma = 1/2$	$\Sigma = -1/2$	$\Sigma = 1/2$	$\Sigma = -1/2$	$\Sigma = 1/2$				
0,0,0	1013	969	991	967	987	-	-	-	-				
0,0,2	1329	1240	1229	1228	1222	1255	1277	1250	1267				
		1255	1266	1253	1260								
1,0,1	1505	1357	1354	1355	1355	1433	1455	1429	1447				
		1420	1442	1416	1432								
		1545	1548	1528	1531								
0,0,4	1645	1509	1500	1496	1494	1520	1492	1499	1492				
		1534	1542	1534	1537					1523	1550	1532	1544
2,0,0	1681	1510	1511	1509	1510	1604	1626	1605	1624				
		1780	1778	1767	1765								
0,1,1	1692	1532	1529	1542	1544	1581	1603	1569	1585				
		1555	1577	1554	1564								
		1840	1844	1875	1880								
1,0,3	1821			1631	1631	1633	1625	1632	1631				
				1692	1682					1707	1729	1703	1718
				1695	1707					1816	1824	1792	1798
				1794	1798								
1,1,0	1868	1665	1668	1674		1773	1795	1767	1768				
		1800	1822	1803									
		2033	2029	2017									
0,0,6	1961	1777	1771	1771		1786	1765	1773	1784				
		1813	1819	1817						1804	1825	1818	1828
0,1,3	2008			1791		1813	1835	1813	1823				
				1793						1851	1841	1855	1854
				1794						2153	2163	2198	2199
				1803									
0,2,0	2055	1828	1832	1875	1880	1924	1946	1917	1933				
		2272	2268	2263	2267								

(a) [Perić et al. 2008]

(b) [Mitić et al. 2016]

# Vibronski spektar $C_5^-$





- Born-Openhajmerova aproksimacija
- *Ab initio* metode
- Mali molekuli
- Rener-Telerov efekat
- Model za tretiranje vibronske sprege kod četvoroatomskih molekula
- Ključan korak - diabatizacija
- Kombinacija neke vrste J-T efekta i R-T efekta
- Model za molekule sa proizvoljnim brojem jezgara

M. Perić, S. Jerosimić, M. Mitić, M. Milovanović, R. Ranković, "Underlying theory of a model for the Renner–Teller effect in tetra-atomic molecules:  $X^2\Pi_u$  electronic state of  $C_2H_2^+$ ", *The Journal of Chemical Physics*, **142**, 174306 (2015), doi: [10.1063/1.4919285](https://doi.org/10.1063/1.4919285).

M. Mitić, R. Ranković, M. Milovanović, S. Jerosimić, M. Perić, "Underlying theory of a model for the Renner-Teller effect in any-atomic linear molecules on example of the  $X^2\Pi_u$  electronic state of  $C_5^-$ ", *Chemical Physics*, **464**, 55 (2016), doi: [10.1016/j.chemphys.2015.11.002](https://doi.org/10.1016/j.chemphys.2015.11.002).

Hvala na pažnji!

