UNIVERSITÉ PARIS SUD

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Univerzalne karakteristike dvodimenzionalnih statistickih modela blizu kriticnih tacaka; pristup Konformalne teorije polja

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Pregled

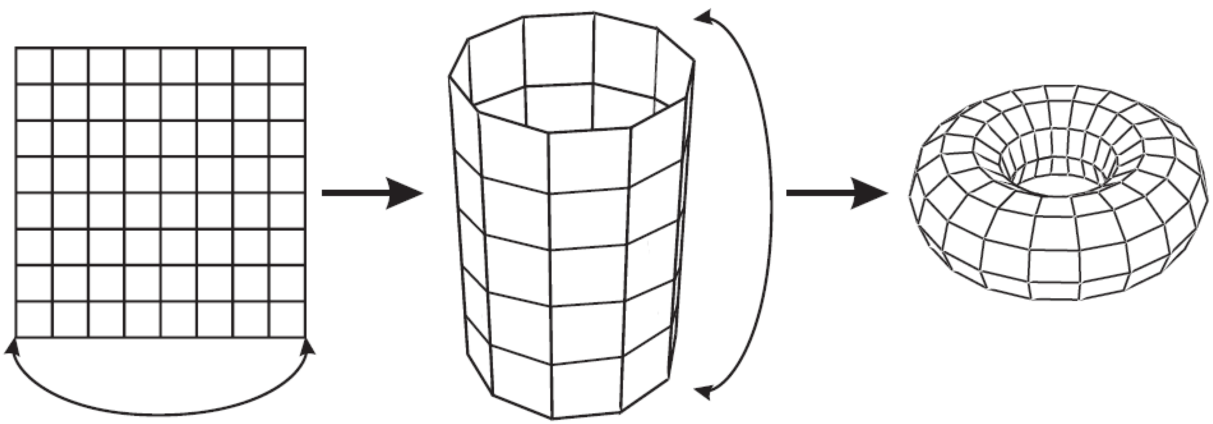
- Izingov model, fazni prelazi
- O konformalnosti
- Posledice konformalnosti
- Metod transfer matrica
- Kasimirov efekat
- Rezultati

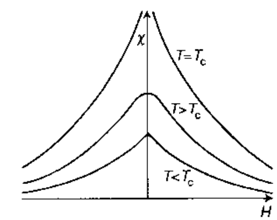
Izingov model

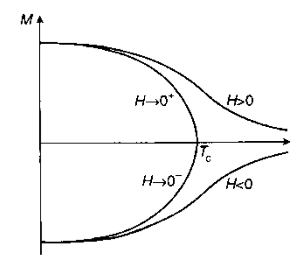
$$H(\{\sigma_{k,j}\}) = -J\sum_{j,k}\sigma_{k,j}\sigma_{k,j+1} + \sigma_{k,j}\sigma_{k+1,j}$$

$$\sigma_{k,L} = \sigma_{k,1} \quad , \quad \sigma_{N,j} = \sigma_{1,j}$$

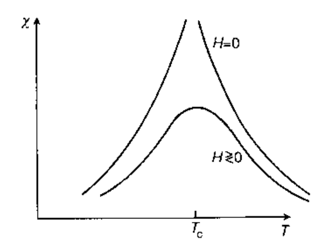
- Model za opisivanje feromagnetizma
- Resetka, cvorevi sa dva stepena slobode {-1,+1}
- U 2D dimenzije resiv, prelaz uocljiv
- Fazni prelaz druge vrste, spontano narusavanje simetrije
- Kriticni eksponenti



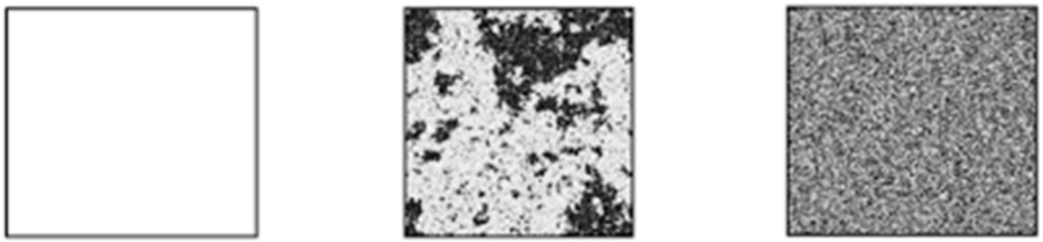




Zero-field specific heat	$C_H \sim \mid t \mid^{-\alpha}$
Zero-field magnetization	$M \sim (-t)^{\beta}$
Zero-field isothermal susceptibility	$\chi_T \sim \mid t \mid^{-\gamma}$
Critical isotherm ($t = 0$)	$H \sim \mid M \mid^{\delta} \operatorname{sgn}(M)$
Correlation length	$\xi \sim \mid t \mid^{-\nu}$
Pair correlation function at T_c	$G(\vec{r}) \sim 1/r^{d-2+\eta}$



MC simulacija



T<Tc   T ~ Tc   T>Tc

Rotacija
Translacija
'zumiranje' – scaliranje

- Scale invariance – Korelaciona duzina divergira i fluktuacije su prisutne na svim skalama
- QFT pristup u kriticnim tackama (bez mase)
- nadogradnja Konformalna simetrija (Belavin, Polyakov i Zamolodchikov)
- Matematicki dokazao: S. Smirnov – Towards conformal invariance of 2D lattice models arXiv: 0708,0032v1

O konformalnosti

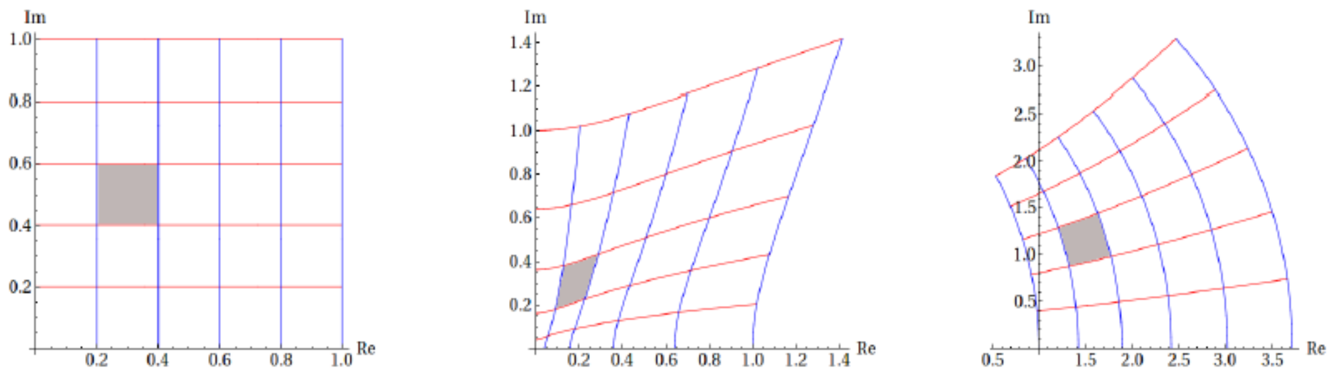


Figure 1: Coordinate transformations. (a) a square lattice in z (b) non-conformal lattice with a map $w(z) = |z|z$ (c) conformal lattice with a map $w(z) = z + e^z$

- Kompleksna ravan, 2D, metricki tenzor

$$x'^{\alpha} = x^{\alpha} + \epsilon^{\alpha}(x)$$
$$\epsilon(x) \ll 1$$

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d}(\partial \cdot \epsilon) g_{\mu\nu}$$

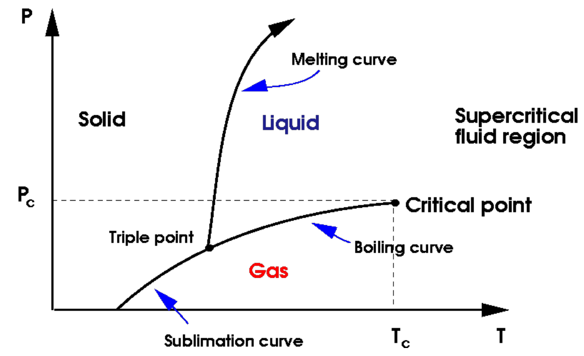
$$g'_{\mu\nu}(x'^{\mu}) = \Omega(x^{\mu}) g_{\mu\nu}(x^{\mu})$$

$d = 1$ $\quad$ $d = 2$ $\quad$ $d \geq 3$.

$$\partial_{\bar{z}} \epsilon = 0$$
$$\partial_z \bar{\epsilon} = 0$$

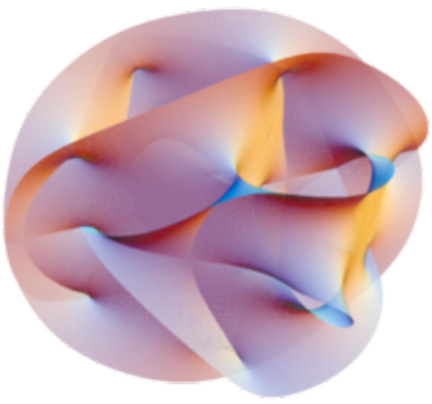
Kosi – Rimanove jednacine

Primene konformalnosti i scale invariantnosti



- Kriticni fenomeni
- Kosmologiji i teoriji gravitacije
- Teoriji stringova

AdS/CFT correspondence



QFT okvir – malo drugaciji recept za CFT

$$\langle \Phi_1 \Phi_2 ... \Phi_N \rangle = \frac{1}{Z} \int \mathcal{D}\phi \Phi_1 \Phi_2 ... \Phi_N e^{-S[\phi]}$$

$$Z = \int \mathcal{D}\phi e^{-S[\phi]}$$

$$\boxed{\delta_\epsilon \Phi(z_i) = \frac{1}{2\pi i} \oint_{C_i} dw \epsilon T(w) \Phi_i(z_i)}$$

$$T_{\overline{z}z} = 0 \qquad T_{z\overline{z}} = 0$$

$$T_{zz} = T(z) \quad , \quad T_{\overline{z}\overline{z}} = \overline{T}(\overline{z})$$

$$T(z) = \frac{1}{2}(T_{00} - iT_{11}) \quad , \quad T(\overline{z}) = \frac{1}{2}(T_{00} + iT_{11})$$

$$T(z)\phi(w,\overline{w}) = \sum_{n=-\infty}^{\infty} (z-w)^{-n-2} L_n \phi(w,\overline{w})$$

Loranovi koeficienti

$$\boxed{\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0} \\ [\overline{L}_m, \overline{L}_n] &= (m-n)\overline{L}_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0} \\ [\overline{L}_n, L_m] &= 0 \end{aligned}}$$

Virasoro algebra – Lie algreba
Pojavljuje se Central charge
Kvantni efekat!

Primarna polja

-Konformalna dimenzija $\Delta = h + \overline{h}.$ $s = h - \overline{h}$

$$\phi(z, \overline{z}) \longrightarrow \tilde{\phi}(w(z), \overline{w}(\overline{z})) = \left(\frac{dw}{dz}\right)^{-h} \left(\frac{d\overline{w}}{d\overline{z}}\right)^{-\overline{h}} \phi(z, \overline{z})$$

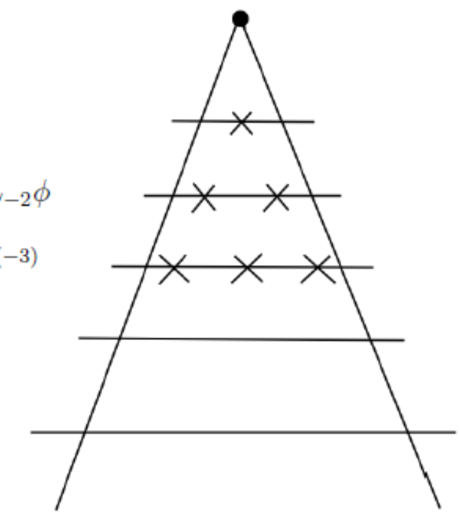
$$\langle \phi_1(z_1, \overline{z}_1) \phi_2(z_2, \overline{z}_2) \rangle = \delta_{h_1, h_2} \frac{C_{12}}{z_{12}^{2h} \overline{z}_{12}^{2\overline{h}}}$$

$$\langle \phi_1(z_1, \overline{z}_1) \phi_2(z_2, \overline{z}_2) \phi_3(z_3, \overline{z}_3) \rangle = \frac{C_{123}}{z_{12}^{a} z_{13}^{b} z_{23}^{c} \overline{z}_{12}^{\overline{a}} \overline{z}_{13}^{\overline{b}} \overline{z}_{23}^{\overline{c}}}$$

Teorija reprezentacija

level	dimension	state
0	Δ	ϕ
1	$\Delta+1$	$\phi^{(-1)} = L_{-1}\phi$
2	$\Delta+2$	$\phi^{(-1,-1)} = L_{-1}^2\phi \quad ; \quad \phi^{(-2)} = L_{-2}\phi$
3	$\Delta+3$	$\phi^{(-1,-1,-1)} \quad ; \quad \phi^{(-1,-2)} \quad ; \quad \phi^{(-3)}$
...	...	...
N	$\Delta+N$	$P(N)$ fields

Verma module



$$L_n\phi = 0 \quad \text{if} \quad n \geq 1$$
$$L_0\phi = h\phi$$

$$L_{-1} = \partial_z$$

$$L_{-2}I = T(z)$$

- Minimalni modeli: konacan broj Verma modula
- Zapravo je to Izingov slucaj

$$I = \phi_{1,1} \quad , \quad \sigma = \phi_{1,2} = \phi_{2,2} \quad , \quad \epsilon = \phi_{2,1} = \phi_{1,3}$$

Transfer matrice

Onsanger

$$Z(L,N) = \sum_{\{\sigma_{j,k}\}} e^{-\beta H(\{\sigma_{k,j}\})}$$

$$H(\{\sigma_{k,j}\}) = -J\sum_{j,k} \sigma_{k,j}\sigma_{k,j+1} + \sigma_{k,j}\sigma_{k+1,j}$$

$$\langle \mu_k | T | \mu_{k+1} \rangle = e^{-\beta E(k;k+1)}$$

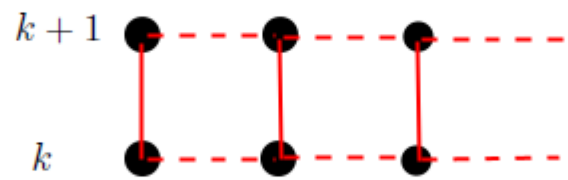
$$E(k;k+1) = -J\sum_{j=1}^{N} \sigma_{k,j}\sigma_{k+1,j} + \frac{1}{2}\left[\sigma_{k,j}\sigma_{k,j+1} + \sigma_{k+1,j}\sigma_{k+1,j+1}\right]$$

$$Z(L,N) = \mathrm{Tr}\left[T_L^N\right]$$

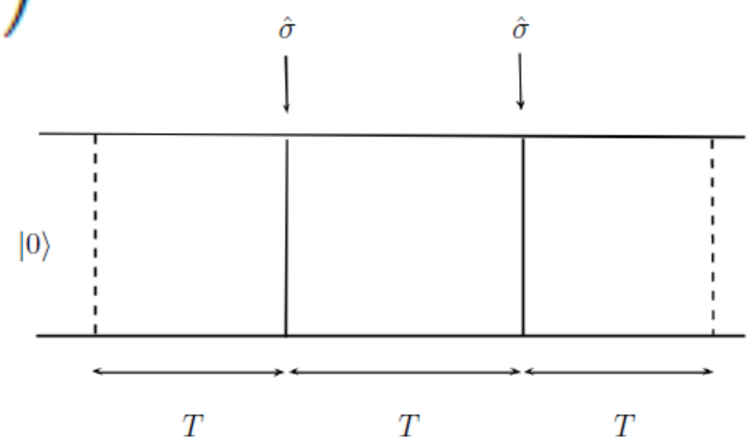
$$| \uparrow\uparrow\rangle, | \downarrow\uparrow\rangle, | \uparrow\downarrow\rangle, | \downarrow\downarrow\rangle$$

$$\boxed{f = -\frac{1}{\beta}\frac{\ln\lambda_0}{L}}$$

$$T = \begin{pmatrix} e^{4J} & 1 & 1 & 1 \\ 1 & 1 & e^{-4J} & 1 \\ 1 & e^{-4J} & 1 & 1 \\ 1 & 1 & 1 & e^{4J} \end{pmatrix}$$



$$\boxed{\langle \sigma_i \sigma_{i+l} \rangle = \lim_{N \longrightarrow \infty} \frac{\mathrm{Tr}\left[T_L^i \hat{\sigma} T_L^l \hat{\sigma} T_L^{N-i-l}\right]}{\mathrm{Tr}\left[T_L^N\right]}}$$

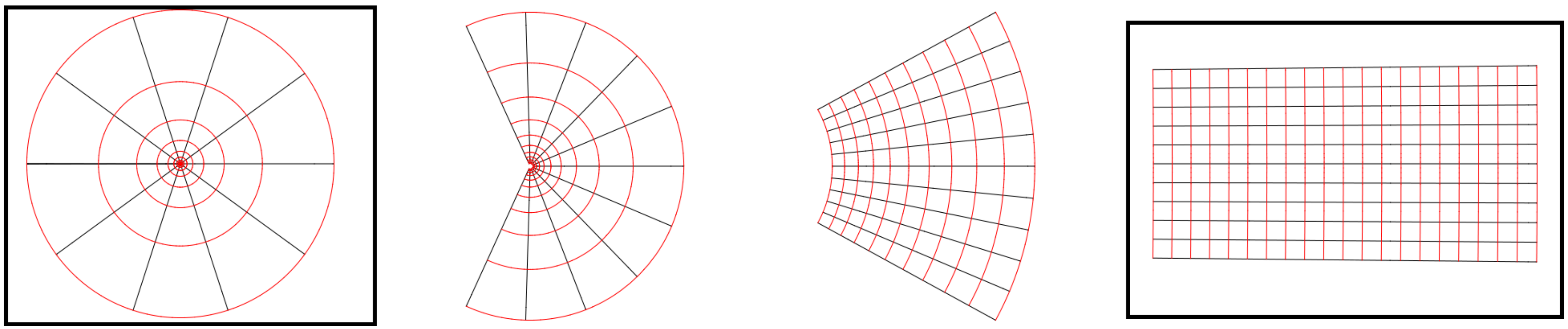


Korespodencija TM i Virasoro operatora

$$T \sim e^{L_0 + \overline{L}_0}$$

$$w = \frac{L}{2\pi} \ln z$$

- dilatation
- Rotation
- Ovde mozda napomenuti v



$$T'(z) = \left(\frac{dw}{dz}\right)^2 T(w) + \frac{c}{12}\{w; z\} \qquad \{w; z\} = \frac{d^3w/dz^3}{(dw/dz)} - \frac{3}{2}\left(\frac{d^2w/dz}{dw/dz}\right)^2$$

$$T_{\rm cyl.}(w) = \left(\frac{2\pi}{L}\right)^2 \left[T_{\rm pl.}(z) z^2 - \frac{c}{24}\right]$$

Primena na Izingov model, slobodna energija, central charge

Analiticka razmatranja u okolini kriticne tacke

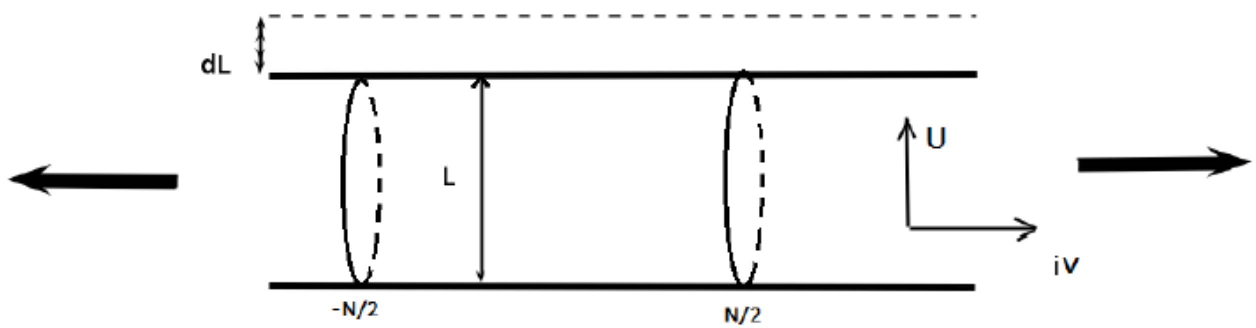


Figure 11: Variation of the width of the cylinder

$$f(L) = f_0 L - \frac{\pi c}{6L}$$

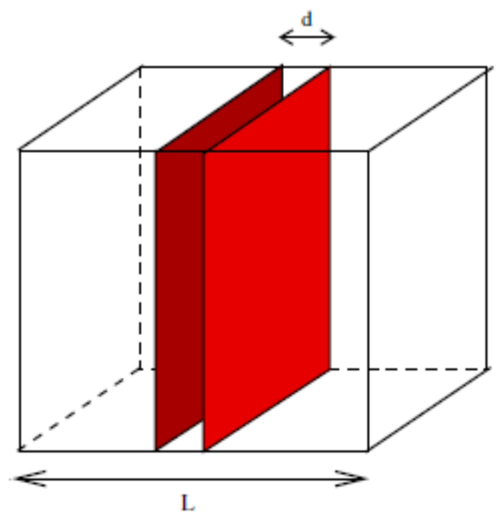
Kazimirov efekat

Central charge se moze posmatrati kao proporcional Kasimirovoj energiji

$$E_{1+1}(d) \;=\; \frac{d}{2\pi a^2} - \frac{\pi}{24d} + \mathcal{O}(a^2)$$

$$f(L) = f_0 L - \frac{\pi c}{6L}$$

Energija vakuuma zbog uvodjenja granicnih uslova,
ide u nulu ako razmak se povecava



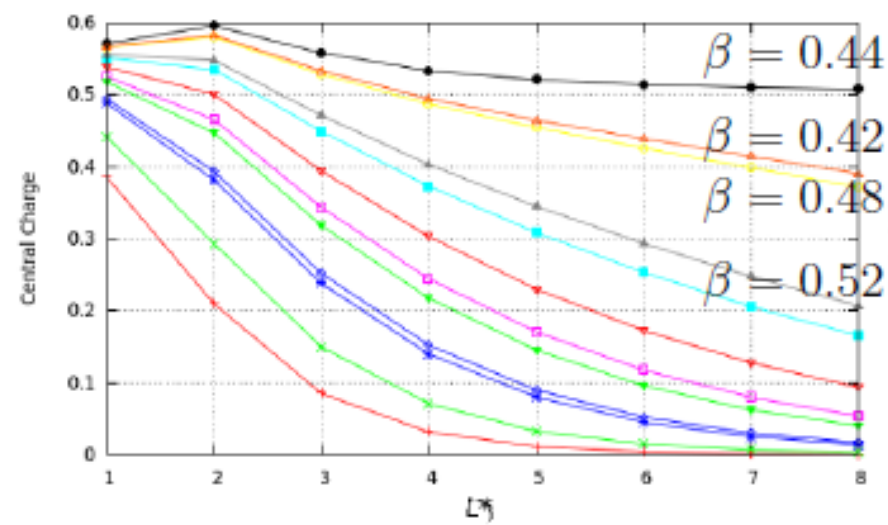
Primena na Izingov model, slobodna energija, central charge

$|\uparrow\uparrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\downarrow\rangle$

$$T = \begin{pmatrix} e^{4J} & 1 & 1 & 1 \\ 1 & 1 & e^{-4J} & 1 \\ 1 & e^{-4J} & 1 & 1 \\ 1 & 1 & 1 & e^{4J} \end{pmatrix}$$

Velicina matrice raste: 2^L

$4 < L < 13.$



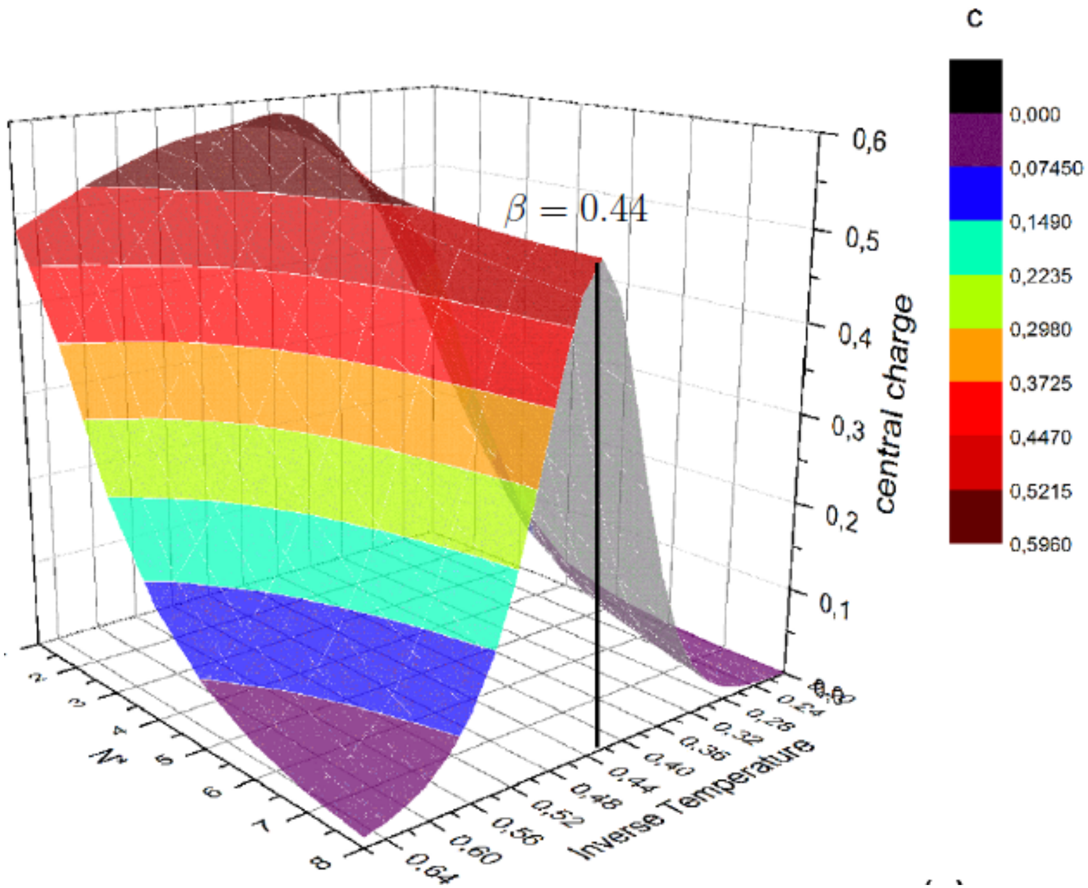
$$f_0(L) = f_0(\infty) - \frac{\pi A e^{-\frac{L}{\xi(\beta)}}}{6L^2} + \frac{A}{L^4} + \cdots$$

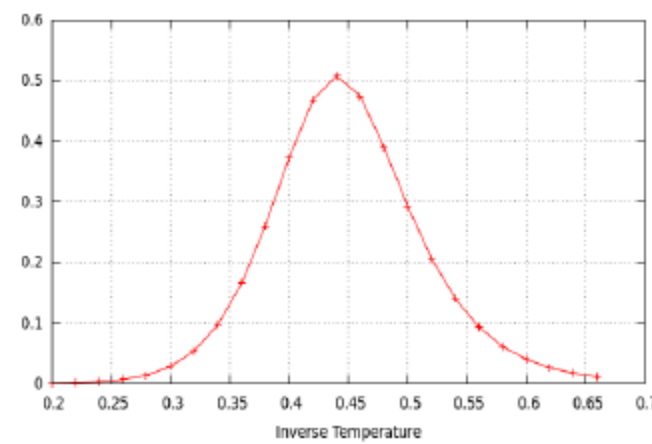
Lanczos algorithm

$$f_0(L) = f_0(\infty) - \frac{\pi c(L)}{6L^2} + \frac{A}{L^4} + \cdots$$

$A = c$

Primena na Izingov model, slobodna energija, central charge





$$\beta_c = -\frac{1}{2}\ln\left(\sqrt{2}-1\right) = 0.440687$$

U blizini kriticne tacke

CFT

$$\langle \phi(w_1)\phi(w_2)\rangle = \left(\frac{dw}{dz}\right)^{-h}_{w=w_1}\left(\frac{dw}{dz}\right)^{-h}_{w=w_2}\langle \phi(z_1)\phi(z_2)\rangle$$

$$\langle \phi(u_1,v_1)\phi(u_2,v_2)\rangle \sim \left(\frac{2\pi}{L}\right)^{\Delta} e^{-\frac{2\pi u \Delta}{L}} \qquad \text{for } u \gg L$$

$$\boxed{f_i(L) - f_0(L) = \frac{2\pi\Delta}{L^2}}$$

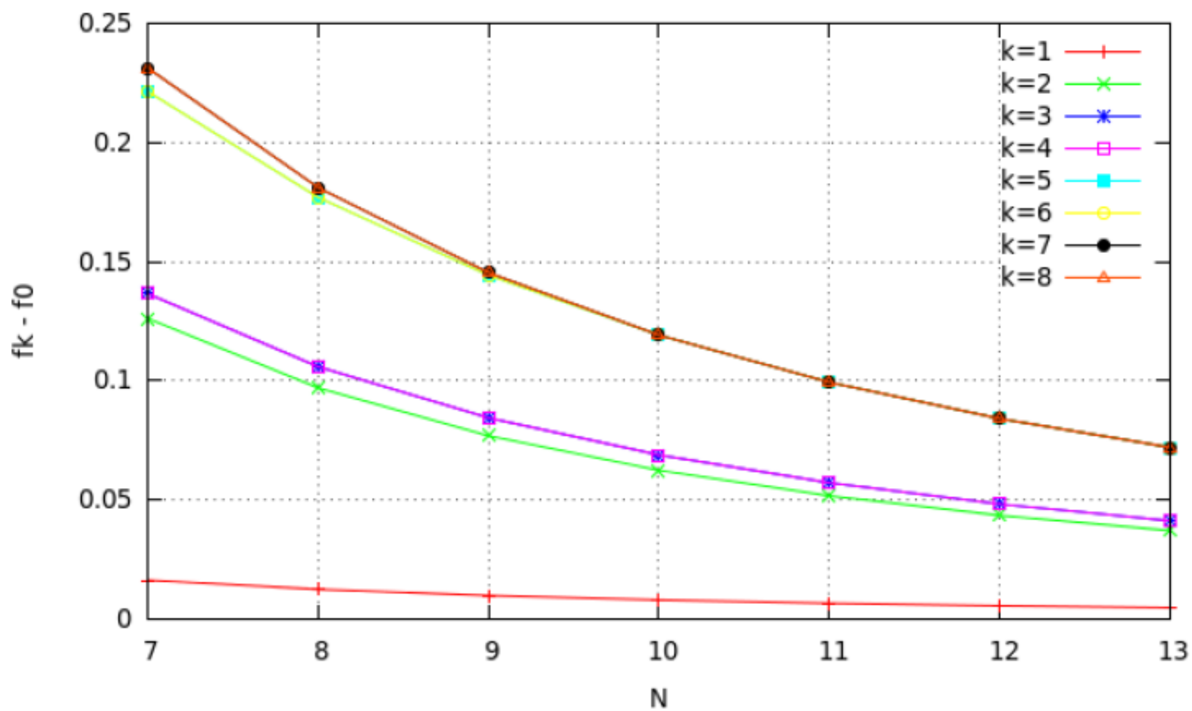
$$f_i(L) - f_0(L) = \frac{2\pi\Delta_i}{L^2} + \frac{B}{L^4} + \ldots$$

TM

$$\langle \phi_i \phi_{i+l}\rangle = \frac{\mathrm{Tr}\left[T^i \phi T^l \phi T^{N-i-l}\right]}{\mathrm{Tr}\left[T^N\right]}$$

$$\langle \phi_i \phi_{i+l}\rangle \sim const. \cdot \left(\frac{\lambda_i}{\lambda_0}\right)^l = const. \cdot e^{l \ln \frac{\lambda_i}{\lambda_0}}$$

$$f_i \cdot L = -\ln\lambda_i$$

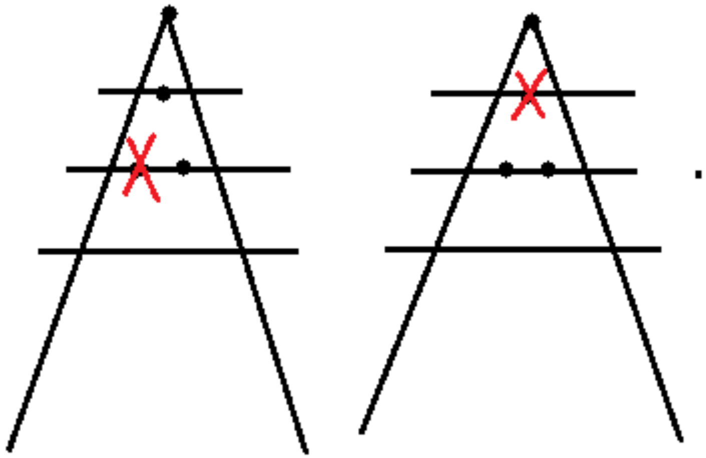


Ublizini kriticne tacke - rezulati

k	Δ	fitting error for a	p	Primary Field
1	0.124954	$\pm 5.091 \cdot 10^{-6}$ (0.004074 %)	0	σ
2	0.999824	$\pm 2.67 \cdot 10^{-5}$(0.00267%)	0	ϵ
3	1.12256	±0.0003538(0.03152%)	±1	σ
4	1.12256	±0.007506(0.03152%)		
5	2.04204	±0.007506(0.3676%)	±1	ϵ
6	2.04204	±0.007506(0.3676%)		
7	1.98511	±0.001975(0.09947%)	±2	I
8	1.98511	±0.001975(0.09947%)		
9	2.01972	±0.01458(0.7218%)	±2	σ
10	2.01972	±0.01458(0.7218%)		
11	2.12011	±0.0007(0.03302%)	0	σ
12	2.91254	±0.02127(0.7301%)	±3	I
13	2.91254	±0.02127(0.7301%)		
14	2.87815	±0.0406(1.411%)	±2	ϵ
15	2.88751	±0.01238(0.4287%)		
16	2.92029	±0.04243(1.453%)	±3	σ
17	2.97181	±0.1223(4.114%)		
18	3.07037	±0.1249(4.068%)	0	ϵ

U blizini kriticne tacke

Energy module Identity module



Critical exponents verified $\eta = 1/4$ $\nu = 1$

$$\Delta^{\sigma} = 0.124987 \sim 1/8$$

$$\Delta^{\epsilon} = 0.999824 \sim 1$$

$$\langle \sigma_0 \sigma_r \rangle \sim \frac{1}{r^{1/4}} \sim \frac{1}{|x_1 - x_2|^{\Delta^{\sigma}}}$$

$$\langle \epsilon_0 \epsilon_r \rangle \sim \frac{1}{r^{\Delta^{\epsilon}}}$$

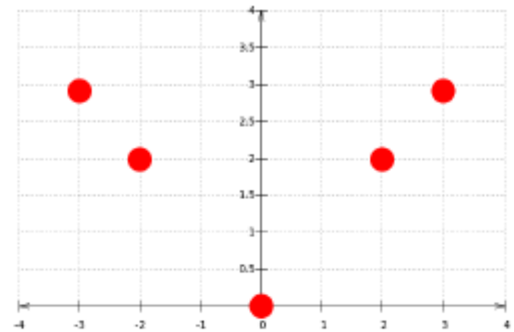


Figure 14: Conformal tower for the identity field I

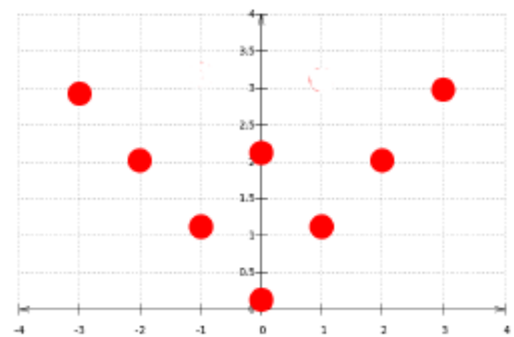


Figure 15: Conformal tower for the identity field σ

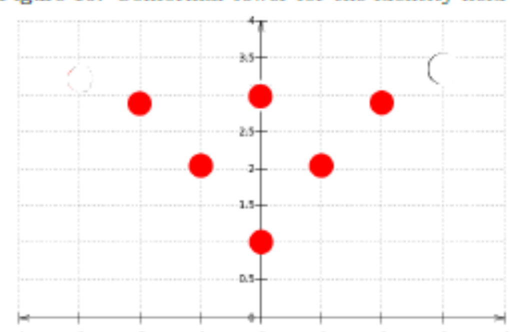


Figure 16: Conformal tower for the identity field ϵ

Trocesticna korelaciona funkcija i Strukturna konstant

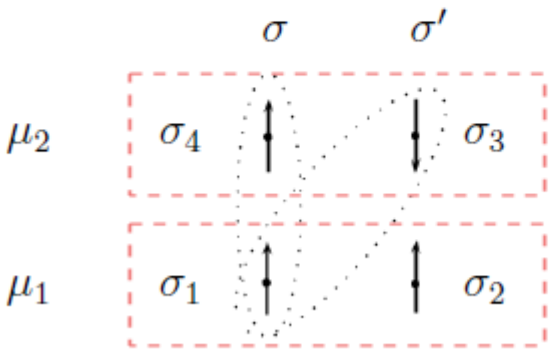
CFT

$$\langle \phi_{h_1}(w_1,\overline{w}_1)\phi_{h_2}(w_2,\overline{w}_2)\phi_{h_3}(w_3,\overline{w}_3)\rangle = \left(\frac{dw}{dz}\right)^{-h_1}\Big|_{w=w_1}\left(\frac{dw}{dz}\right)^{-h_2}\Big|_{w=w_2}\left(\frac{dw}{dz}\right)^{-h_3}\Big|_{w=w_3}\times \frac{C_{\mathbf{1}\ \mathbf{2}\ \mathbf{3}}}{z_{12}^{h_1+h_2-h_3}\,z_{23}^{h_2+h_3-h_1}\,z_{13}^{h_3+h_1-h_2}}\times \text{anti-holomorphic part}$$

TM

$$\boxed{C_{\mathbf{1}\ \mathbf{2}\ \mathbf{3}} = \frac{\langle \phi_1|\hat{\phi}_2(\sigma)|\phi_3\rangle}{\langle \phi_2|\hat{\phi}_2(\sigma)|0\rangle}}$$

Trocesticna korelaciona funkcija i Strukturna konstant



$$\hat{\sigma} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad , \quad \hat{\sigma}' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$C_{\sigma,\sigma,\epsilon} = \frac{\langle\sigma|\hat{\sigma}|\epsilon\rangle}{\langle\sigma|\hat{\sigma}|0\rangle} \quad , \quad C_{\epsilon,\epsilon,\epsilon} = \frac{\langle\epsilon|\hat{\epsilon}|\epsilon\rangle}{\langle\epsilon|\hat{\epsilon}|0\rangle}$$

L	$C_{\sigma,\sigma,\epsilon}$	$C_{\epsilon,\epsilon,\epsilon}$
4	0.434993	0
5	0.455907	0
6	0.468349	0
7	0.476264	0
8	0.48158	0
9	0.485308	0
10	0.488019	0
11	0.490048	0
12	0.491605	0
13	0.492825	0

Zakljucak

- Ukratno prezentovali ideje CFT-a
- Uspostavili korespodenciju TM i CFT-a
- Odredili kriticnu tacku
- Odredili central charge
- Pokazali strukturu Verma modula
- Izracunali strukturnu konstantu (malo drugaciji nacin do sad)

3D Ising ?!?

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Solving the 3d Ising Model with the Conformal Bootstrap II. c-Minimization and Precise Critical Exponents

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Abstract

We use the conformal bootstrap to perform a precision study of the operator spectrum of the critical 3d Ising model. We conjecture that the 3d Ising spectrum minimizes the central charge c in the space of unitary solutions to crossing symmetry. Because extremal solutions to crossing symmetry are uniquely determined, we are able to precisely reconstruct the first several $\mathbb{Z}_2$-even operator dimensions and their OPE coefficients. We observe that a sharp transition in the operator spectrum occurs at the 3d Ising dimension $\Delta_\sigma = 0.518154(15)$, and find strong numerical evidence that operators decouple from the spectrum as one approaches the 3d Ising point. We compare this behavior to the analogous situation in 2d, where the disappearance of operators can be understood in terms of degenerate Virasoro representations.

We also observe mysterious operator decouplings taking place, which may be a hint of the exact solvability of the theory.

https://sites.google.com/site/slavarychkov/

Hvala na paznji!